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HEALTHY AGING AND CAPITAL ACCUMULATION

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Abstract

We study the effects on capital accumulation from public health in an overlapping generations model. Investing in the health of young individuals raises longevity and lowers frailty, influencing capital accumulation through three main channels. First, since it is tax-financed, public health investment reduces disposable income and the capacity to save (cost channel). Second, it prolongs life expectancy, encouraging individuals to save for old age (longevity channel). Third, it reduces frailty and the need to save to finance long-term care (frailty channel). Longevity and frailty have ambiguous effects on taxation when the government subsidizes long-term care. We analytically derive the economic implications of these health channels and numerically illustrate our findings. Our main result is that although public investment in healthy aging is costly, it can stimulate capital accumulation even without directly affecting productivity.

Keywords: Overlapping generations, public health, health channels, capital accumulation.

JEL-Code: H51, I15, O41

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Résumé non-technique

Le souhait de tous est de vivre longtemps et en bonne santé. C'est pourquoi de nombreux gouvernements consacrent des ressources importantes à améliorer la santé publique. Cependant, le vieillissement de la population risque d'augmenter ces dépenses publiques et de mettre à rude épreuve les finances publiques (voir le dernier rapport sur le vieillissement publié par la Commission européenne). Il est dès lors important de comprendre les répercussions économiques des dépenses publiques de santé. Une possible détŕioration de l'état des finances publiques pourrait augmenter le rendement des obligations souveraines exigé par les marchés et, par conséquent, affecter la transmission de la politique monétaire.

Parmi les principales composantes des dépenses de l'Etat pour la santé, on distingue l'investissement dans la santé (comme le dépistage médical, l'immunisation, la réadaptation et les traitements, ...) et les soins pour les personnes dépendantes ("assurance dépendance"). Alors que les soins pour les personnes âgées, qui deviennent dépendantes, sont généralement considérés uniquement comme un coût, les investissements publics dans la santé des jeunes peuvent avoir des bienfaits pour la société, comme l'augmentation de la longévité et la réduction de la fragilité des personnes âgées. Nous nous concentrons sur ces deux effets, qui contribuent au vieillissement en bonne santé (*healthy aging*).[1] Cet aspect des politiques liées au vieillissement est moins étudié, alors que d'autres effets de l'investissement dans la santé sont examinés dans la littérature connexe (p.ex. sur la productivité du travail).

Dans cet article, nous étudions donc les implications macro-économiques de l'investissement public dans la santé à travers ses effets sur la longévité et la fragilité. En particulier, nous considérons la combinaison de ces deux effets liés à la santé sur l'accumulation de capital dans un modèle d'équilibre général à générations imbriquées. Cette approche nous distingue des études qui privilégient un cadre d'équilibre partiel et/ou étudient les effets liés à la santé de manière séparée.

Nous identifions trois canaux par lesquels l'investissement public dans la santé affecte l'accumulation de capital. Premièrement, le coût de l'investissement public dans la santé est générale-

[1] Dans cet article, nous nous focalisons sur l'investissement dans la santé et les soins pour les personnes dépendantes. Ces soins sont en lien direct avec le *healthy aging*, bien qu'il existe d'autres soins de santé comme par exemple les soins ambulatoires.

ment financé par les impôts, ce qui diminue le revenu disponible des ménages et leur capacité d'épargne (*canal coût*). Deuxièmement, l'augmentation de la longévité stimule l'épargne, car les ménages sont incités à épargner plus pour faire face à une vieillesse plus longue (*canal longévité*). Troisièmement, la réduction de la fragilité diminue le besoin de soins pour personnes dépendantes et donc le besoin d'épargner pour subvenir à ces dépenses (*canal fragilité*).

Nous dérivons analytiquement l'impact de l'investissement public dans la santé sur l'épargne et l'accumulation de capital, avant de calibrer notre modèle sur les données de la zone euro, afin de comparer l'importance des trois canaux de transmission. Notre principale conclusion est que l'investissement public dans la santé, bien que coûteux, peut stimuler l'accumulation de capital et donc la croissance même dans un modèle sans lien direct entre santé et productivité.

1 Introduction

Everyone wants to live longer and in good health. Many advanced countries devote considerable public expenditure to promoting healthy aging (Fried et al., 2022; Sowa et al., 2016; WHO, 2014). At the same time, public health expenditures are projected to increase over the coming decades due to population aging, straining public finances. According to the European Commission, public health expenditures in the euro area amounted to 8.9% of GDP in 2022 and are projected to increase by 1.2 percentage points in the next 40 years (EC, 2024).[2] Therefore, it is important to understand the economic implications of public health investment.

Two major components of public health expenditures are health investment and long-term care (LTC). If elderly individuals become dependent, they typically rely on LTC services, which are often modeled simply as costs (Canta et al., 2016). In contrast, health investment in young individuals should improve general health and set the path for healthy aging (Masters et al., 2017; Onofrei et al., 2021).[3] Public health investment is usually tax-financed and therefore reduces disposable income and individuals' capacity to save. However, this cost may be outweighed by the potential benefits of *healthy aging*, which result from increased longevity and reduced frailty.

In this paper, we study the economic implications of public health investment through these two health effects. While there may be other effects of improved health, such as productivity increases (Atolia et al., 2021), we focus on higher longevity and lower frailty, which are directly linked to healthy aging. Raising longevity spurs savings in anticipation of an extended retirement period, which generates greater capital accumulation (Chakraborty, 2004). However, it also extends the period of life spent in dependency (Schünemann et al., 2022). On the other hand, lower frailty reduces the probability of becoming dependent (Marchiori and Pierrard, 2023). Increased longevity and lower frailty thus have opposite effects on the need for long-term care (LTC) and on the savings decisions related to financing these expenditures. Moreover, whether LTC is (partly or entirely) publicly financed matters: a reduction in dependency when

[2]Public health expenditures are the sum of health care and long-term care, two of the aging costs considered in the European Commission's *Ageing Report*.

[3]We use a broad definition of health investment that can encompass any health care contributing to prolonging life and reducing frailty, such as prevention (e.g., vaccinations and screenings), cure, rehabilitation, or medical goods (Wang and Wang, 2021; Jacques and Noël, 2022).

LTC is publicly provided relieves pressure on public finances.

Our paper studies how public investment in healthy aging affects capital accumulation. The related literature typically relies on partial equilibrium models and/or focuses on the impact of either longevity or frailty. A general equilibrium perspective seems more appropriate to account for the direct and indirect implications of these two health effects. We therefore develop a two-period overlapping generations model where the government invests in young individuals' health and (partly) subsidizes LTC costs faced by elderly individuals when dependent. In our model, public health investment leads to higher longevity and lower frailty, which affect individuals' consumption-savings choices.

We analytically derive the impact of public health investment on the level of capital in the steady-state. Three main channels stand out. The *cost channel* implies that tax-financed public health investment directly lowers savings by decreasing disposable income. When LTC is entirely privately financed, health investment stimulates savings and capital accumulation through the *longevity channel* but discourages savings through the *frailty channel* because fewer resources are needed to finance LTC. When LTC is (partly or entirely) publicly financed, an additional effect comes into play that mitigates the effects of the two previous channels. Indeed, higher longevity increases dependency, puts pressure on taxes, and reduces the capacity to save. Lower frailty lessens dependency, relieves tax pressure, and frees resources.

Our theoretical findings are as follows. In the simplest case where frailty and longevity are constant and unaffected by health investment, higher public health investment unambiguously discourages capital accumulation (solely through the cost channel). Savings also decrease when only frailty improves (longevity remains constant) and LTC is privately financed because increased public investment raises taxation (cost channel) and discourages savings (frailty channel) without relieving pressure on public finances. However, in all other cases, public investment has an ambiguous impact on capital accumulation and can spur savings, as illustrated in our numerical analysis.

Calibrating our model to the stylized features of the euro area economy allows us to gauge the strength of the different channels. We find that steady-state capital displays a hump-shaped relationship with public health investment. The longevity channel dominates initially, as public health investment raises steady-state capital, but as public health investment increases further, the cost channel takes over and capital declines. The frailty channel is weak compared to the other two channels. The reason is that frailty only affects LTC costs, which are relatively small if calibrated according to a restrictive definition of LTC, whereas longevity has a direct impact on second period utility in the model. We confirm this reasoning by considering two alternative specifications of the model. Frailty has a more significant impact on steady-state capital when LTC costs are more broadly defined and therefore larger and when frailty has a direct effect on utility (better health raises utility). In this latter specification, savings may increase even if longevity is constant and LTC is entirely privately financed. Our study conveys the important message that public investment in healthy aging may be costly, but it can stimulate capital accumulation even without directly affecting productivity.

Our paper belongs to two branches of the health economics literature concerned with the macroeconomic implications of health investment. The first focuses on a single health effect. Chakraborty (2004) studies how public health investment affects longevity and thereby capital accumulation and growth. Other authors examine the impact of public health spending on labor productivity (Fanti and Gori, 2011; Kuhn and Prettner, 2016). Recent papers focus on the implications of *private* health investment on frailty and LTC expenditure in old age (e.g., Garcia Sanchez et al., 2023; Garcia Sanchez and Pierrard, 2023). Our paper is closer to analyzes centered on the role of the government. Marchiori and Pierrard (2023) analytically identify the welfare-maximizing level of a subsidy to individuals' health investment that reduces frailty and LTC costs. Fabbri et al. (2024) numerically investigate how government investment in public health reduces the dependency rate of the elderly and affects capital accumulation and welfare. Atolia et al. (2021) determine the optimal combination of taxes and subsidies, finding that optimal subsidies increase with the health externality on productivity.[4] Garcia Sanchez et al. (2024) examine the role of health subsidies when longevity and frailty are affected by individuals' health investment decisions, but do not consider the role of capital.

Another branch of the health economics literature considers several health channels, but within a partial equilibrium approach or in models without capital. Schünemann et al. (2022) cali-

[4] Another branch of the health economics literature derives optimal health policy in a partial equilibrium framework and/or does not examine macroeconomic implications (Jack and Sheiner, 1997; Leroux et al., 2011; Jaspersen and Richter, 2015; Canta et al., 2016).

brate a life-cycle model with an ambiguous effect of health improvement on LTC expenditures: longer lifetimes require LTC services until higher ages, while lower frailty diminishes the need for LTC. Menegatti (2014) and Brianti et al. (2018) examine the interactions between individual choices on prevention and cure in a two period framework without capital. These latter approaches disregard the role of the government. Our setting is close to the framework of Garcia Sanchez et al. (2024), where health investment reduces mortality and frailty and affects dependency and LTC costs. However, that paper ignores capital decisions to concentrate on the private health investment choice and derive the optimal subsidies on prevention and LTC. Our study contributes to the literature by examining the health effects on capital accumulation in a general equilibrium model where public health investment lowers mortality and frailty and has counteracting effects on dependency and LTC costs.

The rest of the paper is organized as follows. We present the model in Section 2 and provide some analytical results in Section 3. We discuss the calibration of our model in Section 4 and provide a numerical analysis in Section 5. Section 6 concludes.

2 The Model

This section develops an overlapping generations model with identical individuals who benefit from perfect foresight. They live two periods, working when young (first period of life) and retiring when old (second period of life). The size of the new generation is constant and normalized to 1. An agent born in t inelastically supplies one unit of labor and obtains wage w_t, saves s_t and consumes c_t. Health investment x_t is chosen by the government and financed through wage taxation (Chakraborty, 2004). Health investment contributes to private health capital h_t with health status at old age being a linear function of health investment when young, $h_{t+1} = x_t$, as in Marchiori and Pierrard (2023). The agent survives to the second period with probability $\xi(h_{t+1}) = \xi(x_t) \in [0, 1]$, which is increasing and concave in health investment x_t, i.e. $\xi'(x_t) > 0$, $\xi''(x_t) < 0$ (Heer and Rohrbacher, 2021). In the second period, the agent retires, consumes d_{t+1} and faces long-term care (LTC) costs ℓ_{t+1} with probability $\pi(h_{t+1}) = \pi(x_t) \in [0, 1]$, which is decreasing and convex in x_t (Chakraborty et al., 2016). A fraction θ of these costs is financed by the government.[5]

Households

We assume the representative individual has a second period consumption and a second period budget constraint that are averages over two states: a second period life in bad health needing LTC with probability π and a second period life in good health requiring no LTC with probability $1-\pi$, see Appendix A for a discussion.

The expected lifetime utility of a new-born (representative) agent takes the following logarithmic form

$$U_t = \log(c_t) + \beta\, \xi(x_t) \log(d_{t+1}) \tag{1}$$

where c denotes consumption when young, d consumption when old, $\beta \in (0,1)$ the subjective discount factor, ξ the survival rate and x public health investment.

The young agent faces the following budget constraint

$$c_t + s_t = w_t(1-\tau_t) \tag{2}$$

where s stands for savings, w for the wage and τ for the labor income tax.

In old age, the individual faces long-term care costs ℓ with probability π and a share θ of these LTC costs are publicly financed. The budget constraint of the retired individual born in t is given by

$$\xi(x_t)\,[d_{t+1} + \pi(x_t)(1-\theta)\ell_{t+1}] = R_{t+1}\, s_t \tag{3}$$

The above constraint states that savings during youth (including interest) are allocated to the sum of consumption when old and expected LTC costs, all multiplied by the survival rate. The individual's problem can be described as follows. After inserting (2) and (3) in (1), the individual maximizes her/his lifetime utility by choosing s_t, which leads to

$$d_{t+1} = \beta\, R_{t+1}\, c_t \tag{4}$$

Equation (4) is the Euler equation for consumption, which states that the marginal rate of substitution between current and future consumption equals the expected return on savings.

[5]Note that considering a non-linear relationship between health status, h, and health investment, x, would not change our results as we would just need to adapt the curvature of the functions ξ and π. By assuming a linear relationship between h_{t+1} and x_t, we can focus the rest of the analysis on health investment.

Firms

Labor is normalized to one so k represents both total capital and capital per young individual. The representative firm produces final goods under perfect competition according to a Cobb-Douglas production technology using capital and labor as inputs: $f(k_t) = A\,k_t^{\alpha}$, where A is the productivity level and $\alpha \in (0,1)$ is the capital share in production. Capital fully depreciates after one period. The firm rents capital and labor from households and pays them their respective marginal product

$$R_t = \alpha\,A\,k_t^{\alpha-1} \tag{5}$$

$$w_t = (1-\alpha)\,A\,k_t^{\alpha} \tag{6}$$

Capital accumulation

With a fully depreciated capital stock at the end of each period, savings by the young determine next period's capital stock

$$k_{t+1} \quad = \quad s_t \tag{7}$$

Government

The government finances health investment among the young x and subsidizes a share θ of long-term care expenditures ℓ incurred by those old who become dependent π. x and θ are policy variables. Old age health care costs take the form

$$\ell_t = \mu\,w_t \tag{8}$$

with $\mu > 0$. This functional form implies that health care depends positively on gross wage w. The link between wages and health care reflects the important labor component in LTC services.

The share of the old who become dependent is affected by mortality and morbidity

$$\Lambda(x_t) = \xi(x_t)\,\pi(x_t) \tag{9}$$

Given the properties of $\xi(x_t)$ and $\pi(x_t)$, the impact of health investment on the number of dependent individuals is ambiguous, formally

$$\frac{d\Lambda(x_t)}{dx} = \underbrace{\xi'(x_t)\pi(x_t)}_{\geq 0} + \underbrace{\xi(x_t)\pi'(x_t)}_{\leq 0} \lesseqgtr 0 \tag{10}$$

Indeed, investing in health has an ambiguous effect on the share of dependent people as it extends life duration, see first term in (10), and reduces frailty, see second term in (10). The first effect leads to an increase in the demand for long-term care (Schünemann et al., 2022), while the second effect implies lowers LTC expenditure (Marchiori and Pierrard, 2023). Empirical studies find that if effective measures are focused on diseases that considerably shorten life expectancy then there is an increase in health care spending, while if measures are focused on diseases with a small effect on longevity then health care expenditure declines (see e.g. Grootjans-van Kampen et al., 2014).

The government levies taxes to finance health investment and a share of LTC costs. The budget is balanced in every period through payroll taxes τ_t, which implies[6]

$$x_t + \Lambda(x_{t-1})\,\theta\,\ell_t \quad = \quad \tau_t\,w_t \tag{11}$$

This equation indicates that raising θ induces a direct increase in public LTC expenditures (second term on the left hand side of equation 11). Instaed, increasing public health investment x has an ambiguous impact on total public health expenditures (left hand side). Raising x increases the cost of public health investment (first term on the left hand side) and also affects Λ by extending lifetime and reducing frailty. If the reduction in frailty is stronger than the reduction in mortality, then a higher x reduces the number of dependent individuals and long-term care expenditures. Finally, changes in x and θ activate general equilibrium forces, as higher total public health expenditure affects savings decisions, wages and the tax rate.

3 Analytical results

In this section, we present the transitional dynamics before discussing the steady-state effects a higher health expenditure.

3.1 Transitional dynamics

We characterize the dynamics of the capital stock. Using equations (2) and (3) in (4) gives

$$\frac{R_{t+1}}{\xi(x_t)}s_t - \pi(x_t)(1-\theta)\ell_{t+1} \quad = \quad \beta R_{t+1}\left[(1-\tau_t)w_t - s_t\right] \tag{12}$$

[6] In OECD countries, social security is typically financed through taxes on labor income (social security contributions). Since the labor supply is exogenous in the model, the labor tax does not generate distortions and is equivalent to a lump-sum tax.

Combined with equations (5)-(8), the above equation yields

$$k_{t+1} = \frac{(1-\tau_t)(1-\alpha)Ak_t^\alpha}{1+\frac{1}{\beta\xi(x_t)} - \pi(x_t)(1-\theta)\mu\frac{1-\alpha}{\alpha\beta}} \tag{13}$$

The (direct) effects of health investment on longevity ξ and frailty π appear in the denominator and exert opposing forces on savings. Longer lifetimes encourage savings while lower frailty reduces LTC costs and the need to save. Health investment also influences the budget and thus the tax rate. Using equation (11) leads to capital dynamics of the form $k_{t+1} = z(k_t)$ where

$$z(k_t) = \frac{z_1}{z_2}k_t^\alpha - \frac{x_t}{z_2} \tag{14}$$

$$z_1 \equiv (1-\xi(x_{t-1})\pi(x_{t-1})\theta\mu)(1-\alpha)A > 0$$
$$z_2 \equiv 1+\frac{1}{\beta\xi(x_t)} - \pi(x_t)(1-\theta)\mu\frac{1-\alpha}{\alpha\beta} > 0$$

Note that $z_2 > 0$ is guaranteed if second period consumption is positive.[7] Proposition 1 provides the conditions for the existence and stability of a non-trivial steady-state $\bar{k} > 0$.

Proposition 1 *Define* $\overline{A} \equiv \frac{1}{\bar{z}_1}(\frac{x}{1-\alpha})^{1-\alpha}(\frac{z_2}{\alpha})^\alpha$ *with* $\bar{z}_1 \equiv (1-\Lambda(x)\theta\mu)(1-\alpha)$ *in the steady-state when* $k_{t+1} = k_t = k$ *(and* $x_t = x_{t-1} = x$*).*

$$\text{The economy has} \begin{cases} \text{no steady-state} & \text{if } A < \overline{A}, \\ \text{one stable steady-state} & \text{if } A = \overline{A}, \\ \text{two steady-states} & \text{if } A > \overline{A}, \text{ one being unstable and the other stable.} \end{cases}$$

Proof. We have the following properties of the $z(k_t)$ curve. $z'_{k_t}(k_t) = \alpha\frac{z_1}{z_2}k_t^{\alpha-1} > 0$ and $z''_{k_t}(k_t) = (\alpha-1)\alpha\frac{z_1}{z_2}k_t^{\alpha-2} < 0$ with $\alpha \in (0,1)$. Moreover, $z(0) \leq 0$ as $x_t \geq 0$, $\lim_{k_t\to 0^+} z'_{k_t}(k_t) = +\infty$ and $\lim_{k_t\to+\infty} z'_{k_t}(k_t) = 0$.

Consider the following two conditions:

$$z(k_t) = k_{t+1} \tag{15}$$

$$z'(k_t) = 1 \tag{16}$$

Both conditions hold in equilibrium. The first condition states that the two curves (left and right hand sides of (15)) intersect in equilibrium and the second condition that the curves have the

[7] Since the right hand side of (12) is positive, the left hand is positive as well, which implies $\alpha/(1-\alpha) > \Lambda(x_t)(1-\theta)\mu$ and thus $z_2 > 0$.

same slope in equilibrium. When k_{t+1} is the tangent line to $z(k_t)$, then there is one unique stable steady-state. Using (14) and combining (15) and (16) in the steady-state when $k_{t+1} = k_t = k$ (and $x_t = x_{t-1} = x$), we can eliminate k and obtain

$$\frac{\alpha A \bar{z}_1}{z_2} \left(\frac{\alpha}{1-\alpha} \frac{x}{z_2} \right)^{\alpha - 1} = 1$$

which is equivalent to $A = \overline{A}$. Thus when $A < \overline{A}$, $z(k_t)$ is always below the k_{t+1} line and there is no steady-state. When $A > \overline{A}$, $z(k_t)$ crosses k_{t+1} line twice, due to the above properties of $z(k_t)$. The lower intersection point corresponds to an unstable steady-state and the upper intersection point is the stable steady-state. ■

The rest of the analysis is based on Assumption 1 below, which restricts our attention to the case where there exist two steady-states of which one is non-trivial, stable and positive $\bar{k} > 0$.

Assumption 1

$$A > \bar{A} \tag{17}$$

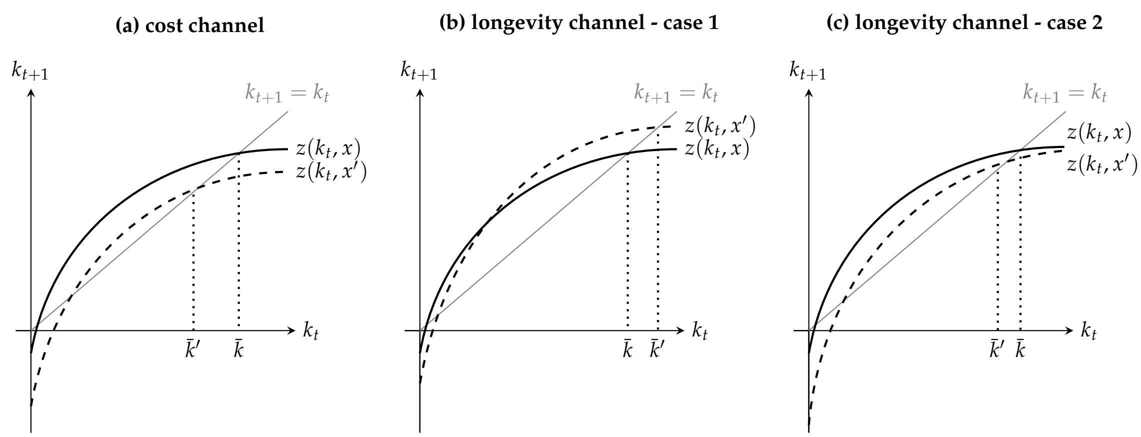
One possible path of the capital stock to its steady-state $\bar{k}$ is depicted by the solid line in Panel (a) of Figure 1. Under Assumption 1, there are two intersections between function $z(k_t)$ and the 45^o degree line. The intersection at the low level of capital represents the unstable steady-state equilibrium. The capital stock is just sufficient to cover health investment and would be zero in case of no health investment. The non-trivial, stable long-run equilibrium is indicated by $\bar{k}$.

3.2 Externality

In our model, public health investment x_t and the public subsidy θ to LTC costs are exogenous, while the labor tax rate τ_t adjusts to balance the public budget in every period. Consider, for the moment, the case where the government fixes the tax at a constant rate $\tau_t = \tau$, and health investment x_t balances the public budget. Abstracting for simplicity from LTC costs ($\mu = 0$) implies that health investment depends on capital investment: $x_t = \tau(1-\alpha)Ak_t^{\alpha}$. In such a setting, capital investment exerts an externality on longevity because households do not internalize the effect of their savings on health expenditure. The effect of this externality appears in equation (13), which resembles findings in Chakraborty (2004) when $\mu = 0$. Higher savings lead to better economic outcomes and provide more resources for health investment, thereby reducing mortality. As individuals expect to live longer, they discount the future less and tend to save more.

Let us go back to our case, where the tax rate balances the public budget as health investment and the LTC subsidy are exogenous (see also de la Croix and Ponthiere, 2010). An externality arises through household choices affecting the tax rate $\tau_t = x_t/w_t + \xi(x_{t-1})\pi(x_{t-1})\theta\mu$. Savings improve economic outcomes (w) and lower the relative cost of health investment (x/w), thereby reducing the tax rate and freeing up resources. Individuals' savings decisions do not fully consider this effect.

Figure 1: Examples of capital dynamics (k) following an increase in x



Implications of raising health investment $x' > x$. $\bar{k}$ is the unique stable long-run equilibrium associated to function $z(k_t, x)$ and $\bar{k}'$ the one associated with function $z(k_t, x')$.

3.3 Health effects

Consider a short-sighted government that engages in an arbitrary level of health investment without considering its effects. We assume that health investment is constant, $x_t = x$ for all t.

We distinguish three effects of health investment on steady-state capital. The first is the *cost channel*, operating through the budgetary cost of health investment. Assuming health investment does not result in health improvements, longevity (ξ) and frailty (π) remain constant. Thus, investing in health only affects the last term in the numerator of equation (14). Figure 1 depicts various possible capital stock dynamics. Increasing health investment ($x' > x$) implies a lower intercept ($-x/z_2$) of the curve $z(k_t)$, leading to a clear downward shift from the $z(k_t, x)$

curve (solid line) to the $z(k_t, x')$ curve (dashed line) as shown in Panel (a) of Figure 1. Therefore, capital converges to a lower long-run equilibrium $\bar{k}' < \bar{k}$.

The second channel is the *longevity channel*, through which health investment increases life expectancy and induces individuals to save more. First, there are no public subsidies to LTC ($\theta = 0$). A higher ξ increases the slope (z_1/z_2) of the curve $z(k_t)$ and further decreases the intercept initiated by the cost channel. The sole effect of the longevity channel is positive, as the numerator of equation (14), $z_1 k_t^\alpha - x_t$, is positive. The total effect on the capital stock depends on the relative strengths of the cost and longevity channels. Capital stock converges to a higher steady-state level if the longevity channel dominates the cost channel, as shown in Panel (b) of Figure 1, and to a lower level if the cost channel is stronger than the longevity channel, as shown in Panel (c) of Figure 1. When LTC costs are publicly subsidized ($\theta > 0$), higher life expectancy increases the number of dependent individuals and therefore LTC expenditure, reinforcing the cost channel. This counteracts the increase in the slope of $z(k_t)$, making it less likely that steady-state capital will be larger with x' than with x when $\theta > 0$ compared to when $\theta = 0$.

Finally, the *frailty channel* captures how health investment improves health status (lower π), reducing demand for LTC and the need for savings to cover these costs. In the absence of the longevity channel ($\xi'_x = 0$) and public LTC costs ($\theta = 0$), a higher x unambiguously reduces the steady-state capital stock (z_1 is constant and z_2 increases, implying that the slope z_1/z_2 and the intercept $-x/z_2$ decrease). The effect of the frailty channel is not depicted in Figure 1, but the outcome is similar to Panel (a) or (c). In the presence of public LTC subsidies ($\theta > 0$), a reduction in frailty may lead to a decrease in the number of dependent individuals and LTC expenditure. This attenuates the cost channel and counteracts the decrease in the slope of $z(k_t)$.

The previous discussion is summarized in the following proposition.

Proposition 2 *The effect of health investment on steady-state capital*

- *is negative when increasing x reduces neither mortality nor frailty ($\xi'_x = 0$ and $\pi'_x = 0$).*
- *is negative when raising x only decreases frailty ($\pi'_x < 0$ and $\xi'_x = 0$) and there are no public subsidies to LTC ($\theta = 0$).*

- *is ambiguous when increasing x only reduces frailty ($\pi'_x < 0$ and $\xi'_x = 0$) and the government subsidizes LTC ($\theta > 0$).*
- *is ambiguous when raising x only reduces mortality ($\xi'_x > 0$ and $\pi'_x = 0$).*
- *is ambiguous when increasing x reduces both mortality and frailty ($\xi'_x > 0$ and $\pi'_x < 0$).*

Proof. See analysis of equation (14) above. ■

4 Calibration

In this section, we present the calibration of the model. We first describe the specifications chosen for the longevity and frailty functions in our model, before discussing the parameter values.

4.1 Functional forms

To simulate our model, we use the following functional forms for longevity and frailty. In line with our discussions in the previous sections, the survival probability function is concave in health investment (Heer and Rohrbacher, 2021)

$$\xi(x_t) = \xi_0 + \gamma \frac{x_t}{1 + x_t} \tag{18}$$

with $\xi_0 \in (0, 1)$, $\gamma \in (0, 1)$, $\xi_0 + \gamma \leq 1$, implying $\xi'(0) \to +\infty$ and $\xi(+\infty) = \xi_0 + \gamma$. ξ_0 stands for health capital at birth and γ represents medical technology influencing how much health investment can raise longevity. Equation (18) is similar to Chakraborty (2004) and is convenient in simulation exercises. The property related to $\xi'(0)$ ensures health investment is positive in equilibrium and the property related to $\xi(+\infty)$ means that γ sets an upper bound on the impact of health investment on longevity.

Moreover, as discussed above, the morbidity probability function (19) below is convex in health investment (Marchiori and Pierrard, 2023; Schünemann et al., 2022), implying that frailty decreases less as health investment increases (see e.g. Chakraborty et al., 2016)

$$\pi(x_t) = \pi_0(1 + x_t)^{-\eta} \tag{19}$$

with $\pi_0 \in (0, 1)$ and $\eta > 0$, implying $\pi(0) = \pi_0$ and $\pi(+\infty) \to 0$.

4.2 Parameter values

The model simulations are based on a stylized calibration of the euro area economy. Table 1 reports the parameter values chosen directly from the literature or the data. The output elasticity of capital is set to the standard value of 0.33. Assuming one generation lasts 35 years, a discount factor (β) of 0.59, corresponding to an annual discount rate of 1.5%, yields a real annual interest rate of 2% (see below).

Let us introduce some definitions (ignoring time subscripts). In the terms of our model, GDP is equal to $y = Ak^{\alpha}$. The GDP shares of public health investment, xy, of public expenditure on long-term care, py and of total expenditure on long-term care, ly, are defined as

$$xy \equiv \frac{x}{y}, \quad py \equiv \frac{\theta \Lambda \ell}{y}, \quad ly \equiv \frac{\Lambda \ell}{y}$$

In the terms of our model, GDP is equal to $y = Ak^{\alpha}$.

We compute health expenditure indicators by averaging across euro area countries over the period 2015-2022 using data from the OECD (2023). The GDP shares of public and total expenditure on long-term care (py and ly) are 1.04% and 1.25%, respectively, which yields $\theta = 0.83$. Public intervention in health care can raise life expectancy and quality of life (Masters et al., 2017; Galvani-Townsend et al., 2022) and includes prevention, curative care and governance (Masters et al., 2017; Onofrei et al., 2021; Wang and Wang, 2021; Jacques and Noël, 2022). Moreover, treatments can be classified in different types of care, for example curative care can combine preventive, rehabilitative and curative care (OECD, 2017). We therefore take a non-restrictive definition for public health investment, including the following OECD categories of (public) health expenditures: curative and rehabilitative care, medical goods, preventive care and governance and health system and financing administration. The GDP share of health investment (xy) is then 5.2% and we can compute the tax rate $\tau = (xy + py)/(1 - \alpha)$. Over 2015-2022, euro area residents aged 64 years had an average life expectancy of 21.7 years (European Commission, 2024), which divided by the length of a generation yields the survival rate in the targeted steady-state $\xi = 0.62$. Among those aged 65 or more, the share of LTC recipients (in institutions and at home) was 12.9% (OECD, 2024), which sets Λ and thus $\pi = \Lambda/\xi = 0.21$ in the targeted steady-state. We can then compute the parameter linking wages to LTC services

$\mu = ly/\Lambda/(1-\alpha)$.

Using (2) and (3) in (4) combined with (5) and (6) yields the steady-state interest factor

$$R = \frac{\beta + \frac{1}{\xi} - \pi\,(1-\theta)\,\mu\,\frac{1-\alpha}{\alpha}}{\beta\,\frac{1-\alpha}{\alpha}\,(1-\tau)}$$

From R, we can compute the annual real interest rate and target it with β. Normalizing $x = 1$, we find $xy/x = Ak^{\alpha} \equiv y$, $k = \alpha y/R$ and $A = y/k^{\alpha}$.

Finally, we compute the values for the parameters characterizing the health functions (18) and (19), i.e. ξ_0, π_0, γ and η. We set ξ_0 and π_0 to match the steady-state values of ξ and π. We next construct historical series on longevity (ξ), frailty (π) and public health investment (x) over the period 2000-2022 and use ordinary least squares to estimate parameter γ from the $\xi - x$ relationship (18) and parameter η from the $\pi - x$ relationship (19), see Appendix B. This procedure yields a point estimate of 0.66 for γ with a 95% confidence interval of (0.59, 0.74) and a point estimate of 1.16 for η with a 95% confidence interval of (1.12, 1.19).

Table 1: Parameter values

	Parameters	Values	Justification/Target
Production	α	0.33	literature
Discount factor	β	0.59	real interest rate
Publicly financed LTC	θ	0.83	public LTC to GDP (py)
Health investment	x	1	normalized
LTC-wage link	μ	0.15	total LTC to GDP (ly)
Productivity	A	13.19	
Longevity	ξ_0	0.29	current longevity
	γ	0.66	$\{\xi, x\}$ relationship
Frailty	π_0	0.46	current frailty
	η	1.16	$\{\pi, x\}$ relationship

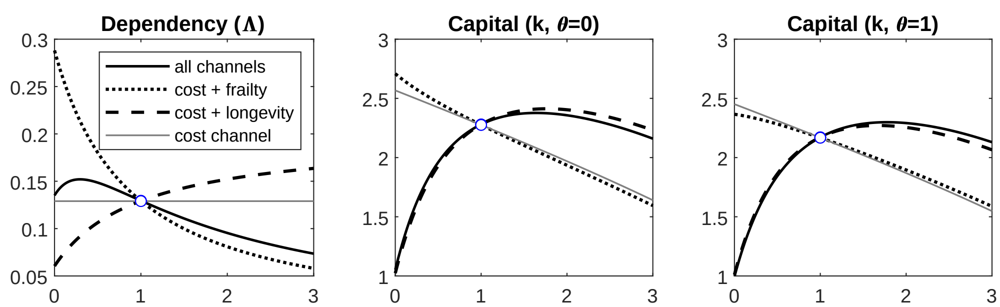
5 Numerical analysis

In this section, we present several numerical exercises focusing on tax-financed increases in health investment (x). The first subsection illustrates the health investment channels discussed in the previous section. The second subsection presents the effects of an increase in health investment on the baseline economy. Finally, the third subsection analyses health effects in alternative specifications of the benchmark economy.

5.1 Health investment channels

Figure 2 highlights the effects of the health channels. The left panel plots dependency (the share of dependent individuals among the elderly) as a function of x. The middle panel plots steady-state capital as a function of x when LTC is entirely privately financed ($\theta = 0$), while the right-hand panel plots steady-state capital when LTC is entirely publicly provided ($\theta = 1$)

Figure 2: Steady-state dependency and capital as a function of x (for $\theta = 0$ and $\theta = 1$)



Steady-state simulations. The circle at $x = 1$ represents the calibrated steady-state. When $\theta = 0$ ($py = 0$) and $\theta = 1$ ($py = ly$), our calibration strategy implies different values for τ and A as well as for R and k. Other parameter values and targets (also μ and ly) are as in the benchmark ($py < ly$) with $x = 1$. Capital is normalized to 1 at its benchmark value when $x = 0$.

Start with the setting where health investment leaves mortality and frailty unchanged (thin gray line, *cost channel*). The share of dependent individuals remains constant as x rises, while steady-state capital decreases. Indeed, an increase in x is financed by wage taxation, which reduces disposable income and savings. Note that steady-state capital is higher with $\theta = 0$ than with $\theta = 1$ for any x. Lower public LTC subsidies force individuals to save more to finance their LTC costs, which also means less taxation and thus a higher capacity to save.

When only mortality is held constant, higher health investment reduces frailty and dependency declines (left panel, dotted line, *cost + frailty*). steady-state capital decreases. Indeed, an increase in x is financed by wage taxation and reduces disposable income. Note that the decrease in steady-state capital as x increases is less pronounced when LTC provision is public

($\theta = 1$) than private ($\theta = 0$). Indeed, when LTC provision is public, lower dependency reduces LTC costs, which has a positive effect on taxation and savings.

When only frailty is constant, higher health investment reduces mortality and leads to increased dependency (left panel, dashed line, *cost* + *longevity*). steady-state capital rises initially as the longevity channel dominates the cost channel. After certain level of x, steady-state capital decreases because the health investment reduces disposable income too much, discouraging savings. As x increases, capital rises less when LTC is fully subsidized ($\theta = 1$) than when there are no subsidies ($\theta = 0$), because higher dependency raises taxation more when $\theta = 1$.

When higher health investment reduces both mortality and frailty, the former effect dominates initially and dependency increases, but eventually the frailty effect dominates and dependency decreases (left panel, thick solid line). steady-state capital rises initially and then decreases after x reaches a certain level.

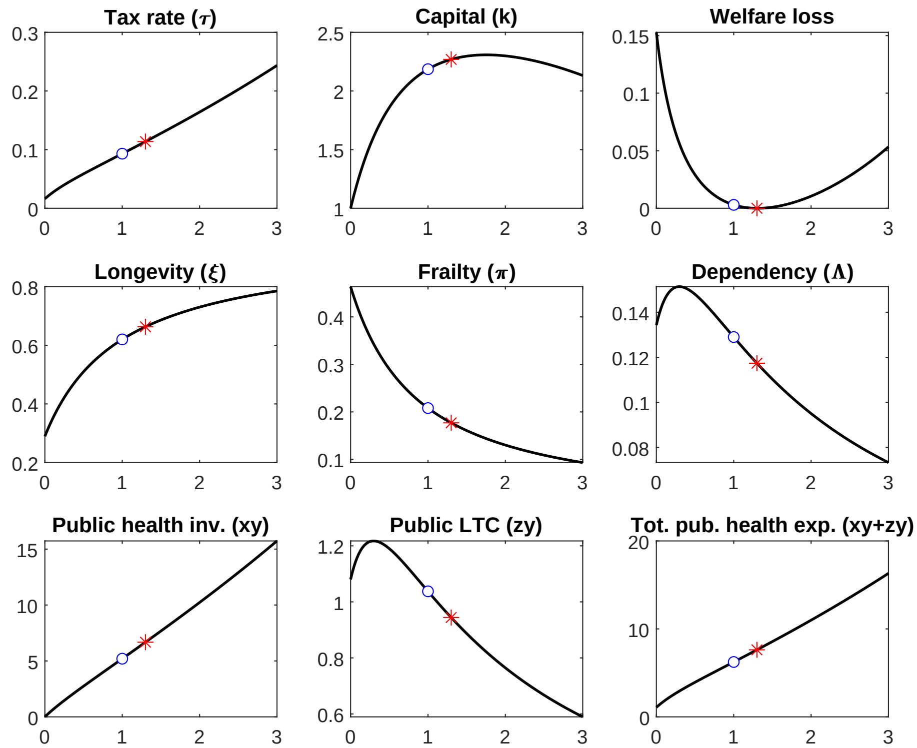
5.2 Effects of health investment on the steady-state

Figure 3 describes the steady-state as a function of health investment (x). Increased health investment x raises longevity and reduces frailty, which has opposing effects on dependency. The share of dependent individuals rises at low levels of x and then declines as x exceeds 0.3, corresponding to a GDP share of health investment (xy) of 1.7%. Public expenditure on LTC follows a similar pattern, initially rising and then falling with dependency. Despite the decrease in the GDP share of public expenditure on LTC (py), the GDP share of public expenditure on health investment (xy) increases, leading to a rise in the GDP share of total public expenditure on health ($xy + py$). Increased health investment x raises the tax rate to keep the budget balanced. However, capital increases due to higher life expectancy, which encourages savings for old age as previously explained.

Welfare of a newborn individual can be defined by the expected lifetime utility in equation (1). Let us consider the change in steady-state welfare, ψ, expressed in terms of consumption, which satisfies

$$\log\left[(1+\psi)c\right] + \beta\,\xi(x)\log\left[(1+\psi)d\right] = U^* \tag{20}$$

Figure 3: Steady-state as a function of health investment x



Steady-state simulations. The circle at $x = 1$ represents the calibrated steady-state and the star indicates the welfare maximizing steady-state. Capital is normalized to 1 at $x = 0$. The GDP share of public expenditure on health investment is denoted xy, that of public expenditure on long-term care is denoted py and that of total expenditure on long-term care is denoted $xy + py$. Welfare loss is expressed as a fraction of lifetime consumption and represents the difference between the individual's utility and the maximum utility.

where the right-hand side U^* represents the maximum expected lifetime utility of a newborn individual, i.e., the utility obtained with the welfare maximizing health investment x^*. The parameter ψ allows to compare an individual's lifetime utility at any value of x with the utility level U^* and represents the fraction of lifetime consumption an individual would gain by living in the optimal policy economy. By construction, ψ is 0 when $x = x^*$ and strictly positive when x is different from x^*. [8]

Figure 3 highlights the welfare-maximizing steady-state (red star). First, we observe that at this

[8]Note that ψ is zero or positive when comparing utility levels within the same model, but it can be negative when comparing utility levels across different models.

point health investment is strictly positive. The welfare loss of living in an economy with no health investment (compared to living in an economy with the optimal x) represents approximately 15.3% of lifetime consumption. Second, we see that the level of health investment in the steady-state calibrated to match observed data (blue circle) is below the optimal level. In our calibration of the model, it would be optimal to increase health investment from the observed level of 5.2% to the welfare maximizing level of 6.7%. This adjustment would improve welfare 0.3% in terms of lifetime consumption.

5.3 Alternative parametrizations

Figure 4 compares the steady-state in the benchmark and in alternative parametrizations. In the benchmark (upper right panel), the frailty channel has a small impact on steady-state capital. This can be observed by comparing the *cost channel* scenario (gray line) to the *cost + frailty channels* scenario (dotted line), as well as when comparing the *cost + longevity channels* scenario (dashed line) to the baseline scenario (black line). The benefits of lower frailty come primarily from a reduction in total LTC costs, which are relatively small in terms of GDP ($ly = 1.25\%$).

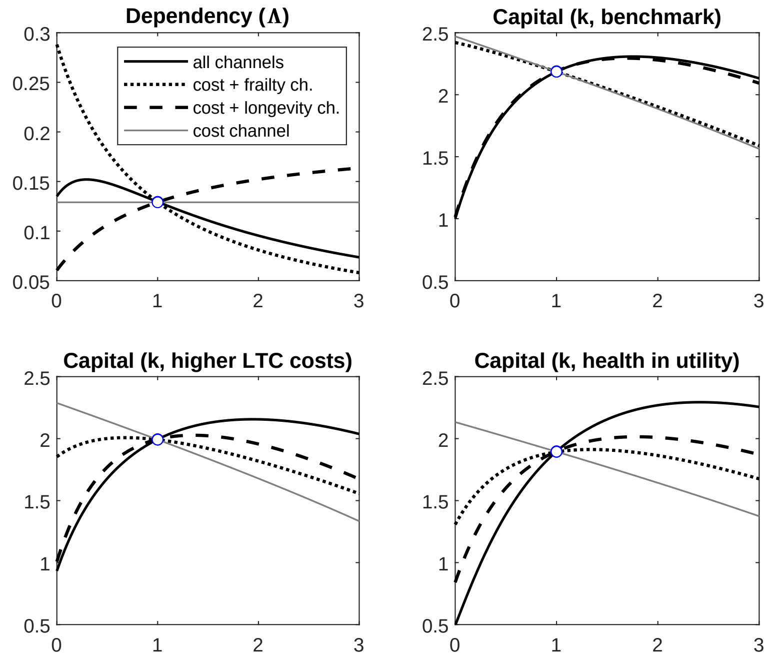
We therefore consider two alternative parametrizations of the model. First, consider an economy where LTC costs amount to 10% of GDP instead of 1.25% as in the benchmark (lower left panel). In this economy, a reduction in frailty has a more significant impact, and there are more pronounced differences between the scenarios. In addition, at low levels of x an increase in health investment can raise steady-state capital even in the *cost + frailty channels* scenario where the longevity channel is switched off.

Second, suppose that individuals value not only the number of years they live but also the quality of health (see discussion in Lang and Rupprecht, 2019). Utility function (1) can be written as

$$U_t = \log(c_t) + \beta\,\xi(x_t)\,(1 - \rho\pi(x_t))\log(d_{t+1}) \tag{21}$$

where $\rho \in [0, 1]$. When $\rho = 0$, we are back to (1) as in our benchmark. When $\rho > 0$, lower frailty raises second period utility because individuals have better health. The lower right panel of Figure 4 presents the different scenarios with "health in the utility" ($\rho = 1$). Again, differences among scenarios are more pronounced than with the benchmark parametrization (upper right panel) and raising x from low levels may increase steady-state capital even in the constant

Figure 4: Steady-state as a function of x with alternative parametrizations



Steady-state simulations. The circle at $x = 1$ represents the calibrated steady-state. Capital is normalized to 1 at its benchmark value when $x = 0$. In the benchmark parametrization, μ is calibrated to match a ratio of LTC to GDP of $ly = 1.25\%$ and $\rho = 0$. The lower left panel "higher LTC costs" reflects higher LTC costs than in the benchmark (10% versus 1.25%) and implies a higher μ (1.16 versus 0.15). The lower right panel "health in utility" ($\rho = 1$) allows frailty to have a direct effect on second period utility.

longevity scenario. Capital dynamics described by (14) are affected as follows:

$$z(k_t) = \frac{z_1}{\tilde{z}_2} k_t^{\alpha} - \frac{x}{\tilde{z}_2} \tag{22}$$

where

$$\tilde{z}_2 \equiv 1 + \frac{z_2 - 1}{1 - \rho\pi(x_t)} > 0$$

Through $\rho > 0$, an increase in x decreases the negative intercept and raises the slope of function $z(k_t)$. In Figure 1, increasing x implies an upward shift in the $z(k_t)$ curve when considering the "health in utility" effect in isolation.

6 Conclusion

This paper examines the effects on capital accumulation from public expenditure on health investment. We develop a general equilibrium model that captures three channels through which public health investment affects savings decisions. Public expenditure on health investment (i) raises taxation, which depresses disposable income (cost channel), (ii) increases average lifetime spurring private savings to finance an extended retirement (longevity channel), and (iii) lowers frailty, reducing the need to save for long-term care (frailty channel). The strength of the longevity and frailty channels depend on how these characteristics affect the dependency rate and long-term care and the extent of government subsidies to the latter, which will affect public finances.

First, we analytically derive the impact of public expenditure on health investment and on private savings and then we calibrate our model to stylized features of the euro area economy, allowing us to gauge the strength of the different channels. For robustness, we examine two alternative parametrizations of our model. Our main finding is that public investment in healthy aging may be costly, but it can stimulate capital accumulation even without directly affecting productivity.

Our work can be extended in two potentially interesting directions. First, considering different types of households could provide further insights. Socio-economic differences such as income or education may determine the share of LTC recipients (Borella et al., 2018). Since low-income households are more likely to be resource-constrained and/or have lower preferences for saving, targeting LTC subsidies to them could enhance overall welfare (Braun et al., 2017). The optimal level of health investment may therefore depend on the socio-economic structure of the population. Second, population aging is expected to raise expenditure on long-term care, and our work could naturally extend to incorporate a detailed demographic structure (see e.g., Bouchet et al., 2017). A large-scale overlapping generations model could account for projected demographic changes and consider various channels in quantifying future health expenditures. This approach could also provide more accurate estimates of the optimal level of health investment.

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A Alternative household perspective

In Section 2, we consider a representative individual who averages over the two possible health states in the second period to obtain a single level of consumption and a single budget constraint in the second period. This is equivalent to analyzing the problem of a representative household with a share π of members requiring LTC and a share $1-\pi$ of members without LTC. The household maximizes the sum of individual utilities and faces a single budget constraint for all its members (perfect insurance among the members). Alternatively, we could consider an individual optimizing over the two possible health states in old age, leading to a different consumption level and budget constraint in each state. This view is presented below and does not change our main conclusions.

Consider the lifetime utility of an individual with a probability π of living the second period in bad health and a probability $1-\pi$ to live the second period in good health:

$$U_t = \log(c_t) + \beta\,\xi(x_t)\log\left[\pi(x_t)d^{\pi}_{t+1} + (1-\pi(x_t))d^{1-\pi}_{t+1}\right] \tag{23}$$

where d^{π} denotes old-age consumption in bad health and $d^{1-\pi}$ old-age consumption in good health.

In old age, the individual faces long-term care costs ℓ with probability π and a share θ of these LTC costs are publicly financed. In the second period, the budget constraint when in bad health is

$$d^{\pi}_{t+1} = \frac{R_{t+1}}{\xi(x_t)}s_t - (1-\theta)\ell_{t+1} \tag{24}$$

and the budget constraint when in good health is

$$d^{1-\pi}_{t+1} = \frac{R_{t+1}}{\xi(x_t)}s_t \tag{25}$$

The individual's problem can be described as follows. Inserting (2), (24) and (25) in (23), the individual maximizes her/his lifetime utility by choosing s_t, which leads to

$$\widetilde{d}_{t+1} = \beta\,R_{t+1}\,c_t \tag{26}$$

where $\widetilde{d}_{t+1} \equiv \left(\frac{\pi(x_t)}{\widetilde{R}_{t+1}s_t-(1-\theta)\mu w_{t+1}} + \frac{1-\pi(x_t)}{\widetilde{R}_{t+1}s_t}\right)^{-1}$ and $\widetilde{R}_{t+1} \equiv \frac{R_{t+1}}{\xi(x_t)}$. Equation (26) is the Euler equation for consumption and the equivalent of (4).

As in Section 3, we can characterize the dynamics of the capital stock as $k_{t+1} = \widetilde{z}(k_t)$

$$\widetilde{z}(k_t) = \frac{\widetilde{z}_1}{\widetilde{z}_2 + \widetilde{z}_1} \left(\widetilde{z}_3 \, k_t^{\alpha} - x_t\right) \tag{27}$$

where

$$\widetilde{z}_1 \equiv \beta \left(\frac{\alpha}{1-\alpha} - (1-\pi(x_t))\xi(x_t)(1-\theta)\mu \right)$$

$$\widetilde{z}_2 \equiv \frac{1}{\xi(x_t)} \frac{\alpha}{1-\alpha} - (1-\theta)\mu$$

$$\widetilde{z}_3 \equiv \left(1 - \xi(x_{t-1})\pi(x_{t-1})\theta\mu\right)(1-\alpha)A$$

Consider $x_t = x$. The analysis of equation (27) leads to the same conclusions described in Propositions (1) and (2). In particular, when $\xi'_x = 0$ and $\pi'_x = 0$, higher health investment leaves $\widetilde{z}_1$, $\widetilde{z}_2$ and $\widetilde{z}_3$ in (27) unchanged and steady-state capital decreases unambiguously through the cost channel. When $\xi'_x = 0$ and $\theta = 0$, a higher x reduces $\widetilde{z}_1$ and steady-state capital diminishes through the frailty channel as well as through the cost channel. In all the other cases, higher health investment has an ambiguous effect on steady-state capital.

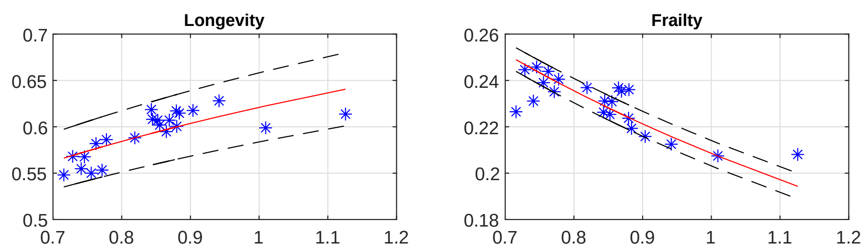
B Parameters of the health functions

This Appendix explains how we set the values for γ, η, ξ_0 and π_0. In short, we regress longevity (ξ) and frailty (π) on health investment (x) to obtain estimates of γ and η and set ξ_0 and π_0 to match the observed values of ξ and π.

First, we explain the construction of the series ξ, π and x based on most recent data (2000-2022). To build the series on x, we combine data on the GDP share of public health investment (xy) described in Section 4 with data on GDP growth (OECD, 2024). Note that the x series is then normalized by its average over the period 2015-2022 (which is the steady-state period in our model calibration). The ξ series is life expectancy among euro area residents at the age of 65 (European Commission, 2024). To obtain the π series, we first calculate the series on dependency (Λ) using data on perceived health status by age (OECD, 2024). More specifically, we calculate a health status indicator equal to one minus the share of the population aged 65+ who report their health to be 'good/very good' since 2000. We then consider the steady-state value for Λ calculated in Section 4 as an end point and let it vary in the past according to this health status indicator to get the Λ series. Dividing Λ by ξ yields the π series.

In a first step, we guess ξ_0 and π_0 and use ordinary least squares to estimate parameter γ from the $\xi - x$ relationship (18) and parameter η from the $\pi - x$ relationship (19). In a second step, we use these parameter values and (18) and (19) to compute ξ and π when $x = 1$ (normalized steady-state value of x). We compare these "fitted steady-states" of ξ and π with their calibrated steady-states (obtained in Section 4). We update ξ_0 and π_0 and loop over steps one and two until fitted and calibrated values of ξ and π are equalized. Figure 5 shows the data points, the regression lines and the 95% confidence intervals for the $\xi - x$ relationship (panel a) and the $\pi - x$ relationship (panel b).

Figure 5: Regression of longevity and frailty on public health investment



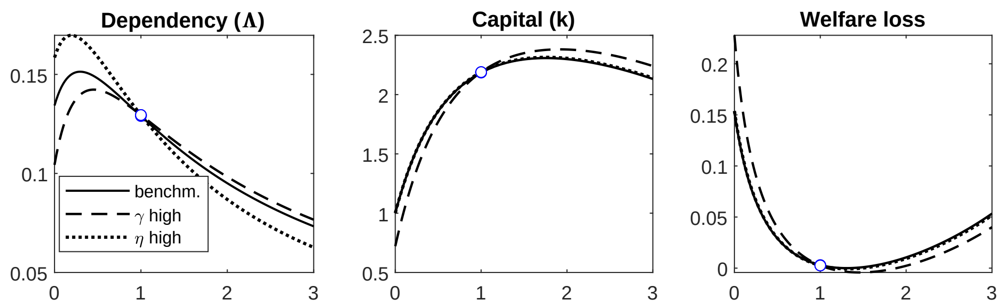
Longevity is the ratio between expected life time at 64 and maximum length of this old-age period. Frailty is the share of dependent individuals among those aged 65 and more. Public health investment is on the x-axis (normalized to one in the steady-state). The red curve is the regression and the two dashed curves delimit the 95% confidence interval.

C Sensitivity analysis

This Appendix examines the robustness of our results first with respect to the parameter values in equations (18) and (19) and with respect to the calibration procedure described in Appendix B.

Figure 6 describes the steady-state effects of health investment for alternative elasticities of the longevity (ξ) and frailty (π) to public health investment (x). We compare our benchmark to two scenarios, one with a higher sensitivity of ξ to x, captured by a larger γ in equation (18), and one with a stronger reaction of π to x, captured by a higher η in equation (18). The calibrated steady-state is the same in the three scenarios. We only comment on the case where $x > 1$, as the explanation is analogous for the case $x < 1$. A larger response of ξ to x implies that the dependency is above the benchmark. As life expectancy is higher, savings are encouraged and capital is larger than in the benchmark, which reduces the welfare loss. When π reacts more strongly to x, then dependency is smaller than in the benchmark (when $x > 1$). This implies smaller LTC costs and thus lower taxes. There is a positive effect on capital and welfare.

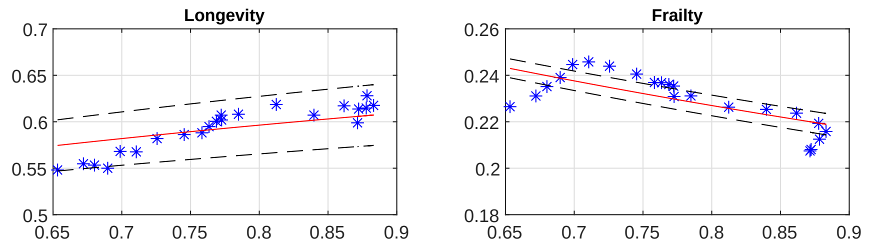
Figure 6: Steady-state for higher γ and higher η



Capital is normalized to 1 at the benchmark value when $x = 0$. In the 'γ high' and 'η high' simulations, γ and η are 20% higher than in the benchmark model. ξ_0 and π_0 are adapted to meet observed values at $x = 1$ in the three models. See text below Figures 2 and 3 for additional information.

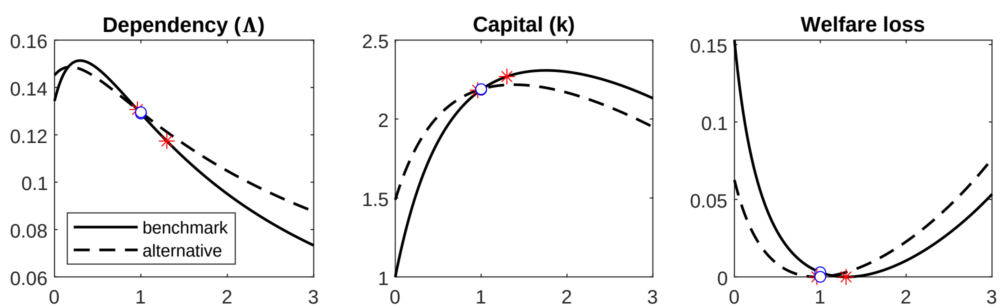
Finally, our strategy described in Appendix B consists in obtaining γ and η from estimations and ξ_0 and π_0 from calibration. Since we consider the steady-state relationships between ξ and π with x, we use contemporaneous values of ξ, π and x in the regressions. Alternative strategies to identify γ, η, ξ_0 and π_0 are possible, such as obtaining the four parameter values from regressions or using lagged values of x in regressions. These strategies do not qualitatively change our findings. For example, Figure 7 shows regressions of ξ and π on a five-year lagged five-year moving average of x leading to $\xi_0 = 0.40$ and $\pi_0 = 0.36$ as well as to $\gamma = 0.44319$ with a 95% confidence interval of (0.37, 0.51) and to $\eta = 0.80$ with a 95% confidence interval of (0.77, 0.83). Figure 7 confirms that our main findings remain unchanged.

Figure 7: Robustness analysis of the regressions of ξ and π on past x



Longevity and frailty in year t are regressed on $t-5$ five-year moving average public health investment. See text below Figure 5 for further details.

Figure 8: Robustness analysis of the steady-state as a function of x



The benchmark simulations are those of Figure 3 and the alternative simulations feature health function parameters based on the regressions described in Figure 7.

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