CAHIER D'ÉTUDES WORKING PAPER

N° 192

FROM BROWN TO GREEN: CLIMATE TRANSITION AND MACROPRUDENTIAL POLICY COORDINATION

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NOVEMBER 2024

EUROSYSTÈME

From Brown to Green: Climate Transition and Macroprudential Policy Coordination*

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November 14, 2024

Abstract

We develop a dynamic, stochastic general equilibrium (DSGE) model for the euro area that accounts for climate change-related risk. The model features polluting ("brown") firms and non-polluting ("green") firms and a climate module with endogenous emissions modeled as a byproduct externality. In the model, exogenous shocks propagate throughout the economy and affect macroeconomic variables through their impact on interest rate spreads. We assess the business cycle and policy implications of transition risk stemming from changes in the carbon tax, and the implications of micro- and macroprudential tools that account for climate considerations. Our results suggest that a higher carbon tax on brown firms dampens economic activity and volatility, shifting lending from the brown to the green sector and reducing emissions. However, it entails welfare costs. From a policy-making perspective, we find that when the financial regulator integrates climate objectives into its policy toolkit, it can minimize the tradeoff between macroeconomic volatility and welfare by fully coordinating its micro- and macroprudential policy tools.

Keywords: climate risk; macroprudential policy coordination; DSGE models *JEL Classification:* E1, E2, O41, Q5, Q58.

^{*}This paper should not be reported as representing the views of the Banque centrale du Luxembourg (BCL) or the Eurosystem. The view expressed are those of the author and may not be shared by other research staff or policymakers at the BCL or the Eurosystem. We thank BCL staff and participants to the ECB 2021 Workshop on Climate Change Impact Assessment and Analytical Methods for comments and feedback. All errors are my own.

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Résumé non-technique

Un aspect fondamental de la stratégie d'atténuation de la crise climatique adoptée dans le pacte vert pour l'Europe « European Green Deal » de l'Union européenne implique la réduction des émissions de gaz à effet de serre (GES) d'au moins 55 % d'ici à 2030 par rapport aux niveaux enregistrés en 1990, puis atteindre la neutralité carbone « net zéro » d'ici à 2050. Le respect de ces standards serait compatible avec la limitation du réchauffement climatique à un niveau nettement inférieur à 2 degrés Celsius par rapport au niveau préindustriel d'ici à la fin de ce siècle et prévenir ainsi la matérialisation des effets potentiellement irréversibles sur les écosystèmes, la santé humaine et les économies.

Pour initier un transfert massif d'investissements de sources d'énergies fossiles vers des énergies renouvelables, l'engagement pris dans le cadre du « Green Deal » nécessite une mobilisation importante de capitaux. La Commission européenne (CE) estime qu'en combinant les sources de financement publiques et privées, une mobilisation de capitaux d'au moins 1.000 milliards d'euros pour les investissements durables est nécessaire au cours de la prochaine décennie. Dans ce contexte, il est vraisemblable que les changements substantiels des politiques, des technologies et des dynamiques du marché, au fur et à mesure que les économies convergent vers la neutralité carbone, amplifieraient l'incertitude quant à l'importance des pertes futures. Ainsi, ces facteurs représentent une source importante de risque de transition qui aurait un impact significatif sur les ménages, les entreprises et le secteur financier. Il y a lieu de rappeler que les expositions du secteur bancaire de la zone euro aux plus grandes industries ayant une empreinte carbone conséquente s'élevaient en 2023 à près de 70 % du portefeuille de prêts aux entreprises.

Bien qu'il existe un large consensus sur l'importance des risques liés au climat pour la stabilité financière, les positions des décideurs publics quant à l'introduction du risque climatique dans la régulation prudentielle sont plurielles. En 2023, l'Autorité bancaire européenne (ABE) a exclu l'introduction à court terme d'un facteur de soutien vert ou d'un facteur de pénalisation brun dans la pondération des risques. Ce refus a été justifié par les défis posés en termes de conception, de calibrage et d'interactions complexes avec les exigences actuelles en matière du pilier 1 des fonds propres. Néanmoins, pour un soutien de la transition verte, tout en veillant à ce que le secteur bancaire reste résilient, l'ABE a présenté des recommandations pour des améliorations ciblées afin d'accélérer l'intégration des risques environnementaux et sociaux dans le pilier 1 des exigences en matière de fonds propres. De même, la Banque centrale européenne (BCE) et le Comité européen du risque systémique (CERS) ont publié en 2023 une stratégie commune globale pour l'UE pour l'adoption d'une régulation macro-micro prudentielle spécifique au risque climatique compte tenu du caractère systémique du risque véhiculé par le changement climatique.

Par ailleurs, le Conseil des gouverneurs de la BCE, dans un communiqué de mars 2024 sur l'examen du cadre opérationnel de la politique monétaire, a déclaré que « la conception d'un nouveau cadre opérationnel pour le pilotage des taux d'intérêt à très court terme visera à tenir compte des considérations liées au changement climatique ».

Bien qu'il soit largement reconnu que les risques financiers liés au changement climatique posent des problèmes micro-macroprudentiels, la recherche académique n'en est qu'à son début. La présente étude vise à contribuer au bourgeonnement de cette nouvelle littérature via la conception d'un modèle d'équilibre général dynamique stochastique (DSGE) pour la zone euro qui incorpore des considérations spécifiques au risque climatique. Le modèle comprend des entreprises polluantes (« brunes ») et des entreprises non polluantes (« vertes »), ainsi qu'une composante (module) climatique avec des émissions endogènes modélisées comme une externalité de sous-produit.

Dans le modèle, les chocs exogènes se propagent dans l'ensemble de l'économie et affectent les variables macroéconomiques par le biais de l'impact des écarts de taux d'intérêt. Nous évaluons le cycle économique et les implications du risque de transition découlant des modifications de la taxe carbone, ainsi que des implications des outils micromacroprudentiels ayant trait au risque climatique. Les résultats obtenus suggèrent que la progression de la taxe carbone sur les entreprises brunes se traduirait par un ralentissement de l'activité économique et par une baisse de la volatilité, en déplaçant les prêts du secteur brun vers le secteur vert et en réduisant les externalités négatives des émissions de carbone (pollution). En outre, elle entrainerait des coûts en termes de bien-être. Cependant, l'intégration par les autorités de supervision des objectifs climatiques dans la conduite de leur politique prudentielle permettrait de réduire ces coûts. A cet égard, la politique optimale, selon nos résultats, consiste en la coordination des politiques micro et macroprudentielles dont l'objectif est de minimiser l'arbitrage entre la volatilité macroéconomique et le bienêtre.

1 Introduction

"Climate change has consequences for us as a central bank pursuing our primary mandate of price stability, and our other areas of competence, including financial stability and banking supervision."

Christine Lagarde, President of the European Central Bank. International Climate Change Conference, Venice, 11 July 2021

A fundamental aspect in the climate crisis mitigation strategy of the European Union's Green Deal involves the reduction of greenhouse gas emissions by at least 55 percent by 2030 compared to 1990 levels and ensure reaching "net zero" by 2050 [\(Allen et al.,](#page-30-0) [2018\)](#page-30-0). Achieving the above targets is paramount to the ability of limiting global warming to well below 2°C relative to pre-industrial levels by the end of the century and preventing severe and potentially irreversible impacts on the planet's ecosystems, human health, and economies.^{[1](#page-5-0)} To unlock a massive shift of investment from fossil fuel to renewable energy sources, the Green Deal commitment requires a sizable capital mobilization. The European Commission estimates that by combining public and private funding sources, a capital mobilization of at least ϵ 1 trillion in sustainable investments is needed over the next decade [\(European](#page-32-0) [Commission,](#page-32-0) [2019\)](#page-32-0). The substantial changes in policies, technologies, and market dynamics as economies shift towards carbon neutrality create financial uncertainty and potential losses. These factors represent a significant source of transition risk that impact households, firms, and the financial sector. In this respect, the banking sector in the euro area is exposed to high-emitters for over 70 percent of the corporate lending portfolio.

Economists generally consider the carbon tax on emitters to be the most economically efficient approach to reducing emissions [\(Akerlof et al.,](#page-30-1) [2019\)](#page-30-1). However, there is a growing consensus that carbon pricing alone cannot fully address the chronic effects of climate change. In addition, the implementation of carbon pricing mechanisms give rise to several challenges, which include asymmetric distributional impacts that disproportionately affect lower-income groups and regions [\(Grainger and Kolstad,](#page-32-1) [2010\)](#page-32-1),

¹"Net zero" is achieved when the amount of greenhouse gases emitted by human activities is equal to the amount removed from the atmosphere through natural processes (like forests and oceans) or technological means (such as carbon capture and storage), resulting in no net increase in atmospheric greenhouse gas concentrations over the reference period.

potential carbon leakages and lack of harmonization [\(Aichele and Felbermayr,](#page-30-2) [2015\)](#page-30-2), and political resistance [\(Blanchard et al.,](#page-30-3) [2023\)](#page-30-3). These considerations make carbon pricing a necessary but insufficient measure. Thus, achieving carbon neutrality hinges on an integrated approach encompassing a set of complementary policies, including monetary and prudential policies within the limits of their mandate. Against this background, policymakers have initiated work to quantify, monitor, manage and mitigate climate-related risks. A prominent example of such an effort is the Network for Greening the Financial System (NGFS), which has been pioneering work for the assessment of climate-related risks and it has been developing reference scenarios to explore plausible pathways for climate policy, technological developments and their economic impacts on socioeconomic variables [\(Bertram et al.,](#page-30-4) [2020\)](#page-30-4). In the same vein, the Financial Stability Board (FSB) has put forward views on the need for tools and policies to sufficiently address systemic risks arising from climate change. It acknowledged how climate systemic risks can give rise to abrupt increases in risk premia across a wide range of assets, altering asset price (co-)movements and amplifying credit and liquidity risks in ways that are hard to predict. In addition, from a financial stability perspective, the FSB's assessment calls for wide policy coordination, as microprudential tools alone may not sufficiently address the cross-sectoral, global and systemic dimensions of climate-related risks, as well as tail risks and the potential for the financial system to amplify their effects [\(FSB,](#page-32-2) [2023\)](#page-32-2).

Despite a broad consensus regarding the relevance of climate-related risk for financial stability, the positions of policymakers on the policy options available to account for climaterelated risks in the prudential framework are a source of debate, which is constantly evolving. In 2023, the European Banking Authority (EBA) ruled out the introduction of a green supporting factor or a brown penalizing factor in the short-term, initially advocated by the European Commission, on the ground that the use of such adjustment factors presents challenges in terms of design, calibration, and complex interactions with the existing Pillar 1 framework. At the same time, the EBA has put forward recommendations for targeted enhancements to accelerate the integration of environmental and social risks across the Pillar 1, in order to support the green transition, while ensuring that the banking sector remains resilient.^{[2](#page-6-0)} In this context, in 2023 the European Central Bank (ECB) jointly with the European Systemic Risk Board (ESRB) released a comprehensive common EU strategy for macroprudential policies to address climate risk, recognizing climate change as a systemic risk and paving the way toward a regulatory framework where microprudential

²[EBA press release of 12 October 2023](https://www.eba.europa.eu/publications-and-media/press-releases/eba-recommends-enhancements-pillar-1-framework-capture#:~:text=The%20Report%20explains%20why%20the,the%20existing%20Pillar%201%20framework.)

and macroprudential policies can complement each other to ensure that the financial system is robust and resilient in the face of climate-related financial risk [\(ESRB,](#page-31-0) [2021\)](#page-31-0). Finally, in its March 2024 statement on the review of the monetary policy operational framework, the Governing Council of the ECB acknowledged that the design of the new operational framework for steering very short term interest rates will incorporate climate change-related considerations into the structural monetary policy operations.^{[3](#page-7-0)}

In light of the above policy debates and the challenges of climate change, there is a pressing need to develop models that help to understand the complex interactions between climate policies, regulatory policies, and other business cycle shocks. Such models are crucial for assessing their impact on macroeconomic variables under different sources of uncertainty and for quantifying the trade-offs associated with different policy measures. Despite the broad recognition that climate change-related financial risks pose micro-macroprudential concerns, analysis and research is at an early stage and the literature is scant. This paper aims to fill this gap. We build on the ongoing policy discussions, and design an environmental, dynamic stochastic general equilibrium (E-DSGE) model that focuses on the interplay between microprudential and macroprudential policies in a context of transition risk and can assess policy implications.^{[4](#page-7-1)} The key contribution of our paper is to show that a financial regulator aiming to account for climate considerations can successfully adopt a combination of borrower-based and capital-based measures to contribute to reducing emissions in the short-term, but with longer-term welfare costs. We show that if the financial authority commits to the climate objective by fully coordinating its micro-macroprudential measures, the financial stability-efficiency trade-off improves.

The remainder of the paper is structured as follows. Section [2](#page-8-0) provides a review of the related literature. Section [3](#page-9-0) introduces the framework of the model and defines the economic agents. Section [4](#page-20-0) provides the quantitative analysis of the model. Section [5](#page-26-0) presents our micro-macroprudential policy experiments and assesses the welfare implications. Section [6](#page-28-0) concludes.

³[ECB Press Release of 13 March 2024 on the review of the operational framework.](https://www.ecb.europa.eu//press/pr/date/2024/html/ecb.pr240313~807e240020.en.html)

⁴Climate-related risks encompass physical risk—such as damage to infrastructure and property from extreme weather events—and transition risk, which stems from the policy, technological, and market changes required to move towards a low-carbon economy. The interconnected nature of these risks can cascade through financial systems, leading to sudden asset revaluations, increased credit risk, and market volatility.

2 Related literature

Recent literature has shown that there are several mechanisms through which financial institutions can affect environmental outcomes, such as by promoting environmental sustainability through bank lending practices and by using green bonds to finance sustainable projects as a way for the financial sector to support the transition to a green economy. For instance, empirical evidence for the bond market has shown that climate-related factors matter for borrowing costs, as firms with high pollution levels generally incur higher capital costs, which appear to be a significant factor in providing incentives for firms to adopt greener practices [\(Bolton and Kacperczyk,](#page-30-5) [2020,](#page-30-5) [2021](#page-31-1) ; [Krueger et al.,](#page-33-0) [2020\)](#page-33-0). In taking climaterelated considerations into account in our DSGE framework, our paper shares many features with canonical medium-scale DSGE models [\(Christiano et al.,](#page-31-2) [2005;](#page-31-2) [Smets and Wouters,](#page-33-1) [2007\)](#page-33-1) and with a growing literature that introduces financial intermediation into wellestablished quantitative macroeconomic frameworks [\(Gertler and Kiyotaki,](#page-32-3) [2010;](#page-32-3) [Gertler](#page-32-4) [and Karadi,](#page-32-4) [2011;](#page-32-4) [De Walque et al.,](#page-31-3) [2010;](#page-31-3) [Brunnermeier and Sannikov,](#page-31-4) [2014;](#page-31-4) [Sims and](#page-33-2) [Wu,](#page-33-2) [2021\)](#page-33-2). Most importantly, our paper relates to a burgeoning macroeconomic literature that attempts to introduce climate-related considerations into standard general equilibrium models. These models belong to different classes, which fundamentally differ in their underlying assumptions and modeling approach. A non-exhaustive classification includes: computational general equilibrium models (CGE), agent-based models (ABM), Integrated Assessment Models (IAM) and more recently DSGE models. Early attempts to integrate climate risk into general equilibrium models largely focused on the impact of environmental policies and carbon taxes. Nordhaus's DICE (Dynamic Integrated model of Climate and the Economy) and the RICE (Regional Integrated model of Climate and the Economy) models, while not DSGE models in the strict sense, laid foundational work by incorporating economic activity and environmental feedback loops. These models paved the way for DSGE models by illustrating the economic costs of climate change and the benefits of mitigation policies [\(Nordhaus,](#page-33-3) [1992;](#page-33-3) [Nordhaus and Yang,](#page-33-4) [1996;](#page-33-4) [Nordhaus,](#page-33-5) [2011\)](#page-33-5) and [Dietz and Stern,](#page-31-5) [2015\)](#page-31-5). [Acemoglu et al.](#page-30-6) [\(2012\)](#page-30-6) introduced a model where innovation in clean technologies reduces the long-term impact of climate change. They emphasized the role of government policy in directing research and development (R&D) towards green technologies. Their model demonstrated that optimal policy could balance economic growth with environmental sustainability. Another strand of literature examines the interaction between climate policies and macroeconomic stability. For instance, [Annicchiarico and Di Dio](#page-30-7) [\(2015\)](#page-30-7) developed a DSGE model incorporating environmental policies such as carbon taxes and subsidies for green investment. Their findings suggest that well-designed environmental policies can enhance macroeconomic stability by reducing the economic volatility caused by climate shocks, while [Van der Ploeg and Rezai](#page-34-0) [\(2021\)](#page-34-0) developed a DSGE model for the optimal carbon pricing with stranded assets. However, none of the existing literature has assessed the role of micro and macroprudential policy and their interaction in the context of climate transition risk.

3 The model

In this section, we introduce the key ingredients of the model. The economy consists of households, labor unions, retailers, polluting ("brown") and non-polluting ("green") intermediate good firms, and banks. A central government conducts fiscal policy, a monetary authority conducts monetary policy and a regulatory authority conducts micro and macroprudential policies. In the model, households derive utility from consumption and from health status, which is positive influenced by their health expenditure and negatively affected by emissions. Finally, households derive disutility from labor, which is supplied to labor unions at a nominal wage rate. Labor unions bundle together household labor supply according to a CES aggregator and provide labor inputs to each intermediate good producer. Households allocate their labor income between consumption of a composite of brown and green goods, a carbon-related consumption tax, health-related expenditure and bank deposits earning interest. Production is horizontally integrated. Intermediate good firms produce output to be sold to sectoral retailers and are subject to a micro-prudential regulatory constraint that governs their ability to issue debt and obtain bank funding for new investment. Retailers transform the intermediate good at no cost into a final consumption good for each sector, in monopolistic competition and staggered price setting. The economy features a pollution externality arising as a byproduct of brown production. The central government sets a carbon tax on firms and levies an environmental tax on households proportional to brown consumption to internalize the negative externality of emissions on aggregate health. We assume that the resulting tax revenues are distributed to green firms in the form of a subsidy to their R&D expenditure of green firms, which boosts endogenous growth and productivity in the emission-free green sector. Hereafter, we denote the variables and parameters related to the brown sector with superscript or subscript "*B*" and those related to the green sector with superscript or subscript "*G*".

3.1 Households

There is a continuum of identical households of measure unity. Preferences are defined over a consumption bundle (\hat{C}_t) , labor supply (N_t) and a health-related indicator (H_t) , according to the following per-period utility

$$
U_t(C_t, C_{t-1}, N_t, H_t) = \left[\frac{v}{1-v} \ln \hat{C}_t + \omega \frac{H_t}{1+v}^{1+v} - \psi \frac{N_t^{1+\eta}}{1+\eta}\right],
$$
(3.1)

where \hat{C}_t follows a preference specification as in [Bouakez and Rebei](#page-31-6) [\(2007\)](#page-31-6) and [Wolff and](#page-34-1) [Sims](#page-34-1) [\(2017\)](#page-34-1), with \hat{C}_t being a composite of brown and green consumption, C_t^B and C_t^G , respectively:

$$
\hat{C}_t = \phi \left(C_t^B - h C_{t-1}^B \right)^{\frac{v}{v-1}} + (1 - \phi) C_t^{G \frac{v}{v-1}}.
$$
\n(3.2)

The parameter ϕ measures the relative weights on brown and green consumption, and $v >$ 0 is a measure of their elasticity of substitution. When $v > 1$, brown and green consumption are utility substitutes; when $v < 1$, they are utility complements. If $v \rightarrow 1$, utility becomes additive separable. The parameter $h \in [0,1)$ is the coefficient governing the intensity of internal habits in consumption, $\psi > 0$ is a scaling parameter for hours worked, ω is the relative importance of health for the household, $N_t = \sum_{j \in \{B,G\}} \int_0^1 w_{j,i,t} L_{j,i,t} dt$ is labor supply across sector *j* to unions indexed by *i*. The parameter $\eta > 0$ is the inverse of the Frisch elasticity of labor supply. To capture the impact of emissions on households, we postulate a negative nexus between health and emissions (see [Coyle et al.,](#page-31-7) [2003,](#page-31-7) [Conceição et al.,](#page-31-8) [2001](#page-31-8) and [Daellenbach et al.,](#page-31-9) [2020,](#page-31-9) *inter alia*, for supporting evidence) and similarly to [Grossman](#page-32-5) [\(2017\)](#page-32-5) and [Halliday et al.](#page-32-6) [\(2019\)](#page-32-6) we assume that household health evolves according to the following law of motion:

$$
H_{t+1} = [1 - \delta(E_t)] H_t + I_t^H,
$$
\n(3.3)

where $\delta(E_t) = \delta_1^E E_t$ captures the impact of of emissions (E_t) on household health, the magnitude of which is governed by the parameters $\delta_1^E > 0$. The stock of emissions evolves as

$$
E_t = (1 - \delta_x) E_{t-1} + X_t, \tag{3.4}
$$

where δ_x is a constant rate of emissions abatement and X_t is the flow of new emissions.^{[5](#page-11-0)} Equation (3.3) captures the fact that health is negatively affected by both the cumulative stock of emissions and the continuous flow of new emissions. As a consequence, the household needs to increase health-related expenditure, I_t^H , in order to restore health status in $t + 1.6$ $t + 1.6$

Each household can smooth consumption through nominal bank deposits (D_t) that pay a gross interest rate, R_t^D . The household is subject to the following nominal budget constraint:

$$
P_t C_t^B (1 + \tau_{c,t}) + P_t C_t^G + D_t + I_t^H = R_{t-1}^D D_{t-1} + \tilde{W}_t N_t + DIV_t - P_t Z_t, \tag{3.5}
$$

where P_t is the price of the consumption good, $\tau_{c,t}$ a tax on the consumption good produced by the brown sector, \tilde{W}_t is the nominal wage rate for supplying labor inputs to labor unions, *DIV^t* denotes net real lump-sum transfers including profits from the ownership of all nonfinancial firms and net worth from exiting intermediaries, government green subsidies, as well as tax transfers to the government; Z_t is a real transfer to new financial intermediaries paid by households upon the entry of new intermediaries. The household maximizes (3.1) subject to the law of motion of their health status (3.3) and the budget constraint (3.5) .

The first-order conditions with respect to brown consumption (C_t^B) , green consumption, (C_t^G) , labor supply (N_t) , deposits (D_t) , health expenditure (I_t^H) , and health status (H_{t+1}) are:

$$
\lambda_t^h (1 + \tau_c) = \frac{1}{\hat{C}_t} \phi \left(C_t^B - h C_{t-1}^B \right)^{-\frac{1}{\nu}} - \beta h \mathbb{E}_t \frac{1}{\hat{C}_{t+1}} \phi \left(C_{t+1}^B - h C_t^B \right)^{-\frac{1}{\nu}}, \quad (3.6)
$$

$$
\lambda_t^h = \frac{1}{\hat{C}_t} \left(1 - \phi \right) \left(C_t^G \right)^{-\frac{1}{v}} \tag{3.7}
$$

$$
N_{j,i,t} = \left(\frac{\lambda_t^h w_{j,i,t}^-}{\psi}\right)^{\frac{1}{\eta}},\tag{3.8}
$$

$$
1 = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} R_t^D, \tag{3.9}
$$

$$
\lambda_t^h = \lambda_{2,t},\tag{3.10}
$$

$$
\lambda_{2,t} = \frac{\omega}{H_{t+1}} \beta \left[1 - \delta \left(E_{t+1} \right) \right] \tag{3.11}
$$

where λ_t^h is the (real) marginal utility of consumption, $\lambda_{2,t}$ is the real marginal utility of

⁵Abatement may stem from several sources, such as improved efficiency in energy saving or carbon-capture technologies. The parameter δ_x broadly captures the different sources of emission abatement.

⁶For the purpose of our analysis we focus on emissions as the only driver of health deterioration.

health, $\bar{w}_t = \tilde{W}/P_t$ is the real wage relevant for the household, $\Lambda_{t,t+1} = \beta \lambda_{t+1}^h / \lambda_t^h$ is the stochastic discount factor, $\Pi_{t+1} = P_t/P_{t-1}$ is the gross inflation rate. Equations [\(3.6\)](#page-11-3), [\(3.7\)](#page-11-3), (3.8) and (3.9) represent, respectively, the marginal utility of brown and green consumption, the labor supply and the Euler equation governing the intertemporal decision between consumption and saving. Equation (3.11) describes how the household must balance the marginal utility of consumption with the marginal utility of health, taking into account the emissions externality on health. The trade-off between health and emissions implies that higher economic activity leads to a higher deterioration rate of the health status, which requires higher health investments to maintain a given level of health.

3.2 Production

There is an investment firm that creates new physical capital and sells it to intermediate good producers, which can be "brown" or "green". The brown firm generates anthropogenic emissions, which we model as a byproduct externality of production in the brown sector. Both firms rely on bank funding to finance the acquisition of productive inputs and face a "loan-in-advance constraint". A continuum of retail firms repackage the intermediate output at no cost and resell it to a final good producer.^{[7](#page-12-0)}

3.2.1 Emission externality

In the spirit of [Heutel](#page-32-7) [\(2012\)](#page-32-7), [Golosov et al.](#page-32-8) [\(2014\)](#page-32-8) and [Barrage](#page-30-8) [\(2020\)](#page-30-8), we model emissions as a negative consequence of economic activity. Therefore, we assume that flow emissions (X_t) and brown economic activity (Y_t^B) are linked by a pro-cyclical emission function:

$$
X_t = \zeta Y_t^B,\tag{3.12}
$$

where $\zeta > 0$ is the emission-to-output ratio (or the emission intensity).^{[8](#page-12-1)} The central government levies a carbon tax, τ_t , that is proportional to emissions, thus levying $\tau_t X_t$ on emitters.

 7 The derivations of the final good producer, the capital good producers and retailers are standard and reported in the Appendix.

⁸See [Kazemzadeh et al.](#page-32-9) [\(2023\)](#page-32-9) for a comparative analysis on the factors influencing carbon emission intensity.

3.2.2 Brown firm

The representative brown firm has a production function that takes brown capital (K_t^B) and labor (N_d^B) $\binom{B}{d,t}$ as inputs:

$$
Y_t^B = A_t \left(u_t K_t^B \right)^{\alpha} \left(N_{d,t}^B \right)^{1-\alpha} \tag{3.13}
$$

where A_t is the aggregate technological shifter, and α is the elasticity of capital. Brown capital accumulates according to the law of motion

$$
K_{t+1}^{B} = (1 - \delta) K_{t}^{B} + \tilde{I}_{t}^{B}, \tag{3.14}
$$

where \tilde{I}_t^B is new investment and δ is the depreciation rate of capital. The brown firm issues debt (B_t) to finance new investment and is subject to a loan-in-advance constraint as in $Sims$ [and Wu](#page-33-2) [\(2021\)](#page-33-2), which postulates that only a fraction $\psi^B > 0$ of investment can be financed by issuing new debt

$$
\Psi^B \tilde{I}_t^B \le Q_t^B \left(B_t^B - \kappa^B B_{t-1}^B \right),\tag{3.15}
$$

where $\kappa^B \in [0,1]$ is a parameter that proxies for the time duration of nominal debt priced at market price Q_t^B , so that $Q_t^B (B_t^B - \kappa^B B_{t-1}^B)$ is the value of new issuance. The brown firm's maximization plan is to maximize profits, subject to the capital accumulation equation (3.14) , the loan-in-advance constraint [\(3.15\)](#page-13-1), the emission function [\(3.12\)](#page-12-2) and the carbon tax $\tau_t X_t$.

The first-order conditions with respect to labor demand N_d^B $d_{d,t}$, brown capital, K_t^B , capital utilization, u_t^B , and investment, \tilde{I}_t^B , are:

$$
\left(p_{w,t} - \tau_t \zeta\right) \left(1 - \alpha\right) A_t \left(u_t^B K_t^B\right)^\alpha \left(N_{d,t}^B\right)^{-\alpha} = w_t^B
$$
\n(3.16)

$$
p_t^{B,k} M_{2,t}^B \delta'(u_t^B) = \alpha (p_{w,t} - \tau_t \zeta) A_t (u_t^B K_t^B)^{\alpha - 1} (N_{d,t}^B)^{1 - \alpha}, \qquad (3.17)
$$

$$
p_t^{B,k} M_{2,t}^B = \mathbb{E}_t \Lambda_{t,t+1} \left[\alpha \left(p_{w,t} - \tau_t \zeta \right) A_t \left(u_t^B K_{t-1}^B \right)^{\alpha - 1} \left(N_{d,t}^B \right)^{1 - \alpha} \right]
$$

+ $\mathbb{E} \Lambda_{t+1} \left[1 - \delta \left(u_t^B \right) \right] n_t^{B,k} M_B^B$ (3.18)

$$
+\mathbb{E}_{t}\Lambda_{t,t+1}\left[1-\delta\left(u_{t+1}^{B}\right)\right]p_{t+1}^{B,k}M_{2,t+1}^{B}
$$
\n(3.18)

$$
\frac{M_{1,t}^B - 1}{M_{2,t}^B - 1} = (\psi^B)^{-1},\tag{3.19}
$$

$$
Q_t^B M_{1,t}^B = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left[1 + \kappa^B Q_{t+1}^B M_{1,t+1}^B \right],
$$
\n(3.20)

$$
\Psi^B p_t^{B,k} \tilde{I}_t^B = Q_t^B \left(B_t^B - \kappa^B B_{t-1}^B \Pi_t^{-1} \right),\tag{3.21}
$$

where w_t^B is real wage, $p_t^{B,k}$ $t^{B,k}$ is the price of new capital. The terms $M_{1,t}^B$ and $M_{2,t}^B$ are auxiliary variables, which equal one when the loan-in-advance constraint is slack.

3.2.3 Green firm

The representative green firm has a production function that takes green capital (K_t^G) and labor $\left(N_d^G\right)$ $\begin{pmatrix} G \\ d,t \end{pmatrix}$ as inputs. In addition, the green firm receives the proceeds from the carbon tax as a production subsidy (J_t) , which takes it as given in the production function with elasticity ξ :

$$
Y_t^G = A_t \left(u_t K_t^G \right)^{\alpha} J_t^{\xi} \left(N_{d,t}^G \right)^{1-\alpha-\xi}, \qquad (3.22)
$$

where A_t is the aggregate technology, α is its elasticity of green capital. At the end of each period, the green firm purchases capital to be used for production in the subsequent period at nominal price, $P_t^{G,k}$ $t^{G,k}$. Physical capital accumulates according to a standard law of motion^{[9](#page-14-0)}

$$
K_{t+1}^{G} = (1 - \delta) \left(K_t^{G} \right) + \tilde{I}_t^{G}.
$$
 (3.23)

The green firm issues perpetual nominal debt (B_t^G) to finance new investment. Denoting with $\psi^G > 0$ the constant fraction of investment that can be financed by the issuance of new debt, the following "loan in advance constraint" holds:

$$
\Psi^G \tilde{I}_t^G \le Q_t^G \left(B_t^G - \kappa^G B_{t-1}^G \right),\tag{3.24}
$$

where $\kappa^G \in [0,1]$ is the decay parameter of coupon payments, which proxies for the time duration of nominal green bonds (B_t^G) with market price Q_t^G , so that $Q_t^G(B_t^G - B_{t-1}^G)$ is the value of new issuance.

The green firm chooses green capital K_t^G , labor demand , N_d^G $\mathcal{F}_{d,t}^G$, capital utilization \mathcal{U}_t^G and

⁹We refrain from making sector-specific assumptions and keep the two production sectors as symmetric as possible (that is, both firms are subject to the same aggregate technology shock and have the same rate of depreciation rate of capital). In this way, we focus on climate and regulatory factors as the source of sectoral differences when capturing the general equilibrium effects of our model.

investment, \tilde{I}_t^G . The first-order conditions in real terms are:

$$
w_t^G = p_{w,t} \left(1 - \alpha\right) A_t \left(u_t^G K_t^G\right)^{\alpha} J_t^{\xi} \left(N_{d,t}^G\right)^{-\alpha} \tag{3.25}
$$

$$
p_t^{G,k} M_{2,t}^G \delta'\left(u_t^G\right) = \alpha p_{w,t} A_t \left(u_t^G K_t^G\right)^{\alpha-1} J_t^{\xi} \left(N_{d,t}^G\right)^{1-\alpha},\tag{3.26}
$$

$$
p_t^{G,k} M_{2,t}^G = \mathbb{E}_t \Lambda_{t,t+1} \left[\alpha p_{w,t} A_t \left(u_t^G K_{t-1}^G \right)^{\alpha-1} J_t^{\xi} \left(N_{d,t}^G \right)^{1-\alpha} \right] + \mathbb{E}_t \Lambda_{t,t+1} \left[1 - \delta \left(u_{t+1}^G \right) \right] p_{t+1}^{G,k} M_{2,t+1}^G
$$
 (3.27)

$$
\frac{M_{1,t}^G - 1}{M_{2,t}^G - 1} = \left(\psi^G\right)^{-1},\tag{3.28}
$$

$$
Q_t^G M_{1,t}^G = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left[1 + \kappa^G Q_{t+1}^G M_{1,t+1}^G \right],
$$
\n(3.29)

$$
\psi^{G} p_{t}^{G,k} \tilde{I}_{t}^{G} = Q_{t}^{G} \left(B_{t}^{G} - \kappa^{G} B_{t-1}^{G} \Pi_{t}^{-1} \right) + G_{t}^{G}, \tag{3.30}
$$

where w_t^G is real wage, $p_t^{G,k}$ $t^{G,k}$ is the price of new capital. The terms $M_{1,t}^G$ and $M_{2,t}^G$ are auxiliary variables, which equal one when the loan-in-advance constraint does not bind. $J_t = \tau_t Y_t^B$ represent the carbon revenues obtained by the central government, which are rebated to the green firm as a R&D subsidy. The firm takes this input as given and employs it in the Cobb-Douglas production function with elasticity $\xi > 0$.

3.3 Banks

There is a continuum of banks indexed by *i*. Their liabilities consist of household deposits (D_t^B) and bank capital (S_t^B) . Their assets consist of holdings of brown and green corporate bonds, L_t^B and L_t^G , respectively, with market price Q_t^B and Q_t^G . The balance sheet of bank *i* at time *t* reads as

$$
Q_t^B L_{i,t}^B + Q_t^G L_{i,t}^G = D_{i,t}^B + S_{i,t}^B.
$$
\n(3.31)

The bank is subject to a capital constraint that endogenously limits leverage derived as a weak contract enforcement problem [\(Gertler and Karadi,](#page-32-4) [2011\)](#page-32-4). It is possible to show that enforcement constraint is given by (see [A.1](#page-35-0) for the derivation):

$$
v_{i,t} \geq \theta_t \left(\rho^B Q_t^B l_{i,t}^B + \rho^G Q_t^G l_{i,t}^G \right), \tag{3.32}
$$

where $v_{i,t}$ is the continuation value of bank *i* at time *t*, and ρ^B , $\rho^G > 0$ are the degree of asset pledgeability, which can also be interpreted as risk weights. The term θ_t follows a macroprudential policy rule defined later in subsection 3.6 . The enforcement constraint (3.32) embodies the notion that brown and green bonds carry different degrees of liquidity (see also [Bernanke and Gertler,](#page-30-9) [1995,](#page-30-9) [Benigno and Nistico',](#page-30-10) [2017;](#page-30-10) [Sims and Wu,](#page-33-2) [2021\)](#page-33-2).

Each period, an exogenous fraction $(1 - \sigma)$ of commercial banks stochastically exits and transfers its net worth to the household. The household replaces the exiting banks with the same number of new banks. The stochastic exit assumption makes banks extra impatient and prevents bank capital accumulating indefinitely.

The objective of a surviving bank in period *t* is to choose its balance sheet variables to maximize expected terminal net worth given the probability $1 - \sigma$ of exiting after $t + 1$. Given the probability of exiting after *j* periods $(1 - \sigma) \sigma^{j-1}$, the commercial bank seeks to maximize the value function:

$$
v_{i,t} = \max (1 - \sigma) \mathbb{E}_t \sum_{j=1}^{\infty} \sigma^{j-1} \Lambda_{t,t+j} s_{i,t+j}^B,
$$
 (3.33)

subject to the enforcement constraint [\(3.32\)](#page-15-0), where $\Lambda_{t,t+j} \equiv \beta \lambda_{t+j} / \lambda_{t+j-1}$ is the household's stochastic discount factor.

It can be shown that the aggregate real bank net worth, $s_t^B = S_t^B/P_t$ (with P_t being the price index) accumulates from retained earnings as

$$
s_{t+1}^B = \sigma \Pi_{t+1}^{-1} Q_t^B l_t^B (R_{t+1}^B - R_t^D) + Q_t^G l_t^G (R_{t+1}^G - R_t^D) + s_t^B R_t^D
$$
(3.34)

where R_t^B and R_t^G are the gross return on the brown and the green asset, respectively. As in [Carlstrom et al.](#page-31-10) [\(2017\)](#page-31-10), we assume that these returns are given by

$$
R_t^B = \frac{(1 + \kappa Q_t^B)}{Q_{t-1}^B},
$$

\n
$$
R_t^G = \frac{(1 + \kappa Q_t^G)}{Q_{t-1}^G}.
$$
\n(3.35)

The bank solves an optimization problem where it chooses the optimal real holdings of brown and green bonds, l_t^B , l_t^G , respectively:

At the symmetric equilibrium, the first-order conditions are:

$$
\frac{\partial \mathbb{L}_t}{\partial l_t^B} : \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} \left[\left(R_{t+1}^B - R_t^D \right) \right] = \rho^B \theta_t \frac{\lambda_t}{1 + \lambda_t},\tag{3.36}
$$

$$
\frac{\partial \mathbb{L}_t}{\partial l_t^G} : \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} \left[\left(R_{t+1}^G - R_t^D \right) \right] = \rho^G \theta_t \frac{\lambda_t}{1 + \lambda_t},\tag{3.37}
$$

where Ω_{t+1} is defined as

$$
\Omega_{t+1} = 1 + \sigma \left(\frac{\partial v_{t+1}}{\partial s_t^B} - 1 \right). \tag{3.38}
$$

Moreover, as we assume that the bank can always access the central bank's deposit facility, in equilibrium the deposit rate equals the policy rate, thus $R_t^D = R_t$. To provide some intuition on the mechanisms at play, it is worth examining the following relationship relating interest the risk premium to the degree of asset pledgeability, which emerges by combining the bank's first-order conditions:

$$
\frac{\rho^B}{\rho^G} = \frac{(R_{t+1}^B - R_t)}{(R_{t+1}^G - R_t)}.
$$
\n(3.39)

The above condition provides a key insight on the relation between interest rate spreads and the enforcement constraint. First, differences in the degree of liquidity determine the presence of a "yield premium" or "excess return" (i.e., risk premium) between the interest rate on corporate bonds and the risk-free interest rate. In fact, as long as ρ^B , $\rho^G > 0$ there exists a risk premium between the corporate asset and the risk-free rate, as it typically emerges in general equilibrium models where borrowing constraints and interest rate spreads lie at the root of business cycle amplification and capital misallocation [\(Kiyotaki and Moore,](#page-33-6) [1997\)](#page-33-6). Second, differences in the degree of pledgeability, or "liquidity risk", determine the existence and the magnitude of the "greenium", the negative differential to maturity between a green asset and a brown asset with otherwise similar characteristics. As (3.39) must hold in equilibrium, for the green and the brown assets to carry the same risk premium and no "greenium" to exist, it would require $\rho^G = \rho^B$.

In line with recent empirical studies showing that climate corporate bonds in Europe are priced at a discount to the same-risk conventional corporate bonds [\(Sergei and Alesya,](#page-33-7) [2022\)](#page-33-7), we assume the assumption below.

Assumption 1. $\rho^G < \rho^B$.

Therefore, the following proposition holds throughout:

Proposition. A negative spread between R_t^G and R_t^B (greenium) arises in equilibrium as ρ^G $<$ ρ *^B holds by Assumption [1.](#page-17-1)*

It can be shown that in equilibrium, the aggregate bank leverage ratio satisfies:

$$
\phi_t = \frac{\rho^B Q_t^B l_t^B + \rho^G Q_t^G l_t^G}{s_t^B} = \frac{\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} R_t^D}{\left[\theta_t - \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^B - R_t^D\right)\right]}.
$$
(3.40)

The above condition links the endogenous leverage ratio to the role of macroprudential policy, where θ*^t* broadly captures the dynamics of counter-cyclical capital requirements, with $\partial \phi / \partial \theta$ < 0 and $\partial \phi / \partial \theta$ > 0 corresponding to a tightening macroprudential policy (higher capital requirements lower leverage) and a loosening macroprudential policy (lower capital requirements relax leverage), respectively.

3.4 Government and climate policy

The central government acts as the fiscal authority. It collects taxes from households and firms, and issues debt B_t^{gov} *t* at market price *Q gov* t^{gov} . In real terms, carbon tax revenues stemming from the household sector are $T_{H,t} = \tau_{c,t} C_t^B$ and those stemming from the corporate sector are $T_{F,t} = \tau_t Y_t^B$. We assume that the government consumes an exogenous and stochastic amount of final output (G_t) . In addition, it transfers corporate carbon tax revenues one-toone to green firms as subsidy (G_t^G) and transfers the revenues from the carbon tax levied on households back to households as subsidy for green consumption (G_t^C) . The government's budget constraint is

$$
P_t G_t + B_{t-1}^{gov} + P_t G_t^G + P_t G_t^C = P_t T_{H,t} + P_t T_{F,t} + Q_t^{gov} \left(B_t^{gov} - \kappa B_{t-1}^{gov} \right). \tag{3.41}
$$

The left-hand consists of government spending, nominal debt issuance, green subsidies expenditure and household subsidies to green consumption. The right-hand consists of the carbon tax revenues and nominal coupon payments on issued debt. For the government spending, G_t , and government debt, B_t , we assume they follow an exogenous $AR(1)$ process.

3.5 Central bank and monetary policy

The central bank controls the risk-free interest rate according to the Taylor principle [\(Taylor,](#page-33-8) [1993\)](#page-33-8) with interest rate smoothing:

$$
R_t = (R_{t-1})^{\phi_r} \left(R \left(\frac{\Pi_t}{\Pi} \right)^{\phi_{\pi}} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right)^{(1-\phi_r)} \varepsilon_{m,t}^{\sigma_m}, \tag{3.42}
$$

where R_t is the policy rate and R its steady state level, ϕ_r is the coefficient on the interest rate smoothing, ϕ_{π} is the inflation coefficient, ϕ_{y} is the output coefficient, and $\varepsilon_{m,t}$ is a monetary policy shock with standard deviation σ_m .

3.6 A climate-related macroprudential policy rule

Following ongoing policy discussions on a macroprudential approach to climate-related risks, we propose a macroprudential rule by which the financial authority aims to maintain financial stability by also addressing systemic risks arising from climate-related risks.^{[10](#page-19-1)} In this respect, we postulate a formulation that targets credit and emissions dynamics, such as:

$$
\ln(\theta_t) = (1 - \rho_\theta) \ln(\theta) + \rho_\theta \ln \theta_{t-1} + \theta_e (\ln(X_t) - \ln(\bar{X})) + \sigma_\theta \varepsilon_{\theta, t},
$$
(3.43)

where θ_e represents the share of capital requirements that depends on environmental factors expressed as the deviation of emissions from a target ("emission gap"), and $\varepsilon_{\theta,t}$ is a shock with standard deviation σ_{θ} .

3.7 Exogenous shocks

The economy is subject to a technology shock and a shock to the corporate carbon tax. For these shocks we assume $AR(1)$ processes:

$$
\ln(A_t) = +\rho_A \ln A_{t-1} + \sigma_A \varepsilon_{A,t},
$$

$$
\ln(\tau_t) = (1 - \rho_{\tau}) \ln(\tau) + \rho_{\tau} \ln \tau_{t-1} + \sigma_{\tau} \varepsilon_{\tau,t},
$$

 10 [Jawadi et al.](#page-32-10) [\(2024\)](#page-32-10) adopt a similar approach for monetary policy by the Taylor rule to also consider both physical and transition risks.

where $\rho_A, \rho_{\tau} \in (0,1)$ are persistence parameters, and $\varepsilon_{A,t}, \varepsilon_{A,t}$ are the technology shock and the shock to the carbon tax rate, with standard deviations σ_A, σ_{τ} , respectively.

4 Quantitative analysis

4.1 Calibration

The model parameters are set to match key quarterly features of the euro area (EA) over the period 2013Q1-2023Q4. Data are drawn from the ECB Statistical Database Warehouse. For the climate-related block of the model, we use IPCC Assessment Reports, NGFS scenarios, International Energy Agency (IEA) publications and academic literature. By focusing on the zero inflation steady state, we have $\Pi = 1, \Pi^* = 1, /v^p = 1, v^w = 1, w^* = w$. This also means that investment adjustment costs in steady state are irrelevant. From the household block we obtain $\Lambda = \beta$, and $R^D = 1/\beta$. In line with the average bank deposit interest rate over our benchmark period, we target $R^D = 1.0050$, which implies a subjective discount factor $\beta = 0.9950$. The habit formation parameter is set to $h = 0.8$ as in [Gerali et al.](#page-32-11) [\(2010\)](#page-32-11). We set the health parameter in the utility function to $\omega = 0.2984$ to normalize health status in the steady state to one.

Next we target the spreads. We target the brown risk premium to 2.0% to match average yields of 10-year maturity BBB-rated corporate bonds, as they account for about 60% of the total investment-grade corporate bond market in Europe. This implies a return on brown bonds of 2.5%. We set the "greenium" at -0.02%, in the range of values found in [Pietsch and Salakhova](#page-33-9) [\(2022\)](#page-33-9), which implies that the green risk premium is 1.8%, which implies a steady state return on green bonds of 2.3%. This calibration gives a ratio of "risk weights" $\rho^B/\rho^G = 1.28$. Thus we set $\rho^B = 0.99$ and $\rho^G = 0.77$, which captures the idea that households demand higher pledgeability for brown bonds relative to green bonds. We set the physical capital depreciation rate $\delta = 0.025$ on a quarterly basis to match an annual rate of capital depreciation of 10% and $\alpha = 1/3$ as standard in the literature [\(Smets and Wouters,](#page-33-10) [2002\)](#page-33-10). We set the elasticity of the R&D input to $\xi = 1\%$ to remain conservative on the actual availability of green technology. This approach is consistent with the assumptions underlying NGFS scenario narratives. The parameter on the disutility of labor is set to $\psi = 0.953$ and the Frisch elasticity is set to $\eta = 1$ to normalize aggregate labor supply in the steady state to one. The elasticity of substitution parameters are set to $\varepsilon_P = \varepsilon_w = 11$ to match steady state price and wage markups of 10% as in [Quint and Rabanal](#page-33-11) [\(2014\)](#page-33-11). Parameters governing

firms' loan in advance constraints are set in the baseline calibration to $\psi^B = \psi^G = 0.8$, in the range of values used by $Sims$ and Wu [\(2021\)](#page-33-2). The monetary policy rule calibration follows conventional values adopted in the literature, with $\rho_r = 0.8$, $\phi_\pi = 2.5$, and $\phi_\nu = 0.12$, which are in the range of values found in [Gerali et al.](#page-32-11) [\(2010\)](#page-32-11) and [Quint and Rabanal](#page-33-11) [\(2014\)](#page-33-11) suggesting a high interest rate inertia, a strong response of inflation and a weaker response to output. As for the banking sector, the survival rate of bankers is set to $\sigma = .095$, the value set by [Gertler and Karadi](#page-32-4) [\(2011\)](#page-32-4). The steady state ratio between system-wide bank exposures and commercial banks net worth is set to 5, which implies banks' capital requirements are well above regulatory levels in Basel III. On the climate module, we set the emission intensity ζ to match the observed emissions to output ratio of 0.025% quarterly. To calibrate emissions and the emission target, we consider the NGFS orderly scenario. Therefore, the average quarterly reduction needed to achieve a 50% reduction in emissions by 2030 is approximately 2.85%, which we use to set emission target X^* . The parameter capturing the rate at which the stock of existing emissions decay, δ_x , is pinned down by combining the emission function and the law of motion for the stock of emissions. In particular, it gives $E/Y^B = \zeta/\delta_x$. After manipulation and using known steady state values, we find a very small abatement rate for the stock of existing emissions, $\delta_x = 0.00031\%$, in line with our conservative assumptions on limited availability of carbon-capture technology. Following a sequential approach, we can turn to setting the parameter governing the impact of emissions on household health. In this respect, data from the World Health Organization (WHO) show that about 20% to 30% of respiratory diseases globally are attributed to outdoor air pollution. Therefore, we set the coefficient $\delta_1^E = 0.30$ and obtain a steady state ratio of health investment to health status of 0.25. Finally, in the macroprudential policy rule, the coefficient θ_e turns out to be a free parameter as steady-state flow emissions are zero. Therefore, we set it to $\theta_e = 0.05$ to capture a mild increase of capital requirements adjusting for climate risk. Table [1](#page-22-0) summarizes the calibration. Table [3](#page-23-0) reports the business cycle properties of the model.

Parameter	Description	Value
α	Elasticity of output to capital	0.330
β	Household subjective discount factor	0.995
ĸ	Duration parameter on bonds	0.975
h	Coefficient on consumption habits	0.800
ω	Health related parameter	1
ψ_B	Brown firm LTV	0.800
Ψ_G	Green firm LTV	0.800
Ψ	Scaling parameter on disutility of labor	0.953
κ_c	Capital adjustment cost	2.000
δ_1	Intercept parameter on capital utilization costs	0.030
δ_{2}	Slope parameter on capital utilization costs	0.01
η	Frisch elasticity	1.000
ϕ_π	Taylor rule parameter on inflation	2.500
Z	Lump sum transfer to new entering commercial banks	0.05
ρ_m	Persistence parameter of monetary policy shock	0.800
δ_1^E	Emission impact on health parameter	0.30
σ_{m}	Standard deviation monetary policy shock	0.005
ρ_A	Persistence parameter of technology shock	0.900
σ_{A}	Standard deviation technology shock	0.010
ρ_{θ}	Persistence parameter of credit shock	0.900
σ_{θ}	Standard deviation credit shock	0.010
ρ^G	Pledgeability parameter on green bond	0.77
ρ^B	Pledgeability parameter on brown bond	0.99
ε_p	Elasticity of substitution for consumption goods	11
ε_{w}	Elasticity of substitution for labor services	11
ϕ_p	Price stickiness parameter	0.75
ϕ_w	Wage stickiness parameter	0.750
σ	Survival rate of bankers	0.95
$\delta_{\rm r}$	Emission abatement rate	0.0000031
ζ	Carbon intensity	0.00025
X^{\ast}	Emission reduction target rate	0.0285
ĸ	Coupon bond parameter	0.975
$\theta_{\scriptscriptstyle\rho}$	Macroprudential policy rule parameter	0.05

Table 1: Parameters

Variable	Description	Model		Data	
	Key ratios	Mean	St. Dev.	Mean	St. Dev.
Y	GDP	1.000	1.000	1.000	1.000
C/Y	Consumption	0.3350	0.0113	0.4581	0.0215
I/Y	Investment	0.3013	0.0198	0.2588	0.0434
L/Y	Total bank credit	0.5010	0.0119	0.6702	0.0635
X/Y	Flow emission-to-output ratio	0.0056	0.0001	0.0003	0.0001
φ	Leverage ratio banking system	4.8491	0.1514	5.5506	0.2334
	Key rates (gross, $\%$)				
R^D	Bank deposit rate	1.0050	0.0065	1.0065	0.3200
R^B	Brown bond rate	1.0205	0.0082	1.0250	0.6900
R^G	Green bond rate	1.0171	0.0085	1.0230	0.6901

Table 3: Business cycle properties

4.2 Impulse response functions

In this subsection, we provide the model results conditional on the realization of a variety of shocks. All impulse response functions are expressed in terms of the percentage deviation from the steady state. Interest rates and inflation are annualized.

To provide a general intuition of the mechanisms at work, we first report the impulse response functions of key macroeconomic variables conditional on the realization of a onepercent positive shock to total factor productivity (TFP) under different regimes of transition risk. To capture transition risk over the business cycle, we shut-off the dynamic rule for the corporate carbon tax, setting the tax to constant values capturing a regime of low or high corporate carbon regime, $\tau_t = \tau_{Low}$, τ_{High} . Solid lines denote a regime of low transition risk where the corporate carbon tax is set to the baseline level with low transition risk, $\tau_{\text{low}} = 5\%$. In contrast, dotted lines denote a regime of a higher transition risk, where the corporate carbon tax rate is increased to $\tau_{\text{high}} = 25\%$. Figure [4.1](#page-25-0) shows that, as standard, a positive technology shock generates expansionary effects in aggregate variables such as output, consumption, investment and credit. In fact, the positive technology shock raises factors demands due to increasing aggregate productivity. Firms would like to increase investment, thus the demand for external funding increases in order to finance new capital

acquisition. Banks accommodate the higher demand by increasing holdings of both brown and green corporate bonds, although at different intensities due to general equilibrium effects. Therefore, credit supply increases. The shock pass-through exerts upward pressures on corporate bond prices, compressing yields on firm debt and risk-premiums. Brown and green firms demand more credit and increase bond issuance. Banks accommodate higher demand for credit by purchasing more brown and green bonds. However, bank demand for brown bonds is higher than for green bonds due to the higher rate of return carried by holding the brown asset. Therefore brown bond prices increase more than green bond prices causing brown bond yields to compress more than green bond yields. Therefore, the "greenium" increases.

This mechanisms is also amplified by our Assumption [1](#page-17-1) via the enforcement constraint as different degrees of asset pledgeability affect the magnitude of the greenium. Compressing yields on corporate debt relax firm's financing constraints and reduce borrowing costs which, in turn, decrease bank financial intermediation margins. The expansionary credit cycle triggers higher counter-cyclical capital buffers, lowering leverage. On the monetary policy side, inflation declines due to higher firm productivity, lower marginal costs and price stickiness. The central bank reacts to disinflation by lowering the policy rate. Under the assumptions of our model, the carbon tax is generally able to dampen business cycle volatility. As a result, flow emissions decrease, while health status increases reflecting the positive effects of lower emissions.

These results rest on key assumptions. In particular, the presence of a carbon tax on brown consumption that is transferred to households as a "tax credit" for the purchase of green consumption goods, and revenues that finance subsidies to R&D expenditure by green firms. These assumptions are key in explaining the underlying amplification mechanisms of this model, and the particularly desirable ability of shifting lending from the brown sector to the green sector following the realization of a positive technology shock.

4.3 Micro-Macroprudential policy coordination

To better understand the implications of micro-macro prudential policy over the climate transition, we define four scenarios of micro-macroprudential policy coordination following a shock to the corporate carbon tax rate. In this model, we refer to "coordination" as the process by which the financial regulatory authority aligns its micro and macroprudential policies when accounting for climate change-related considerations. Therefore, conditional

Figure 4.1: Impulse response functions of key variables conditional on the realization of a positive technology shock for different degree of the carbon tax τ.

on the realization of a transition risk triggered by a positive shock to the corporate carbon tax, the financial authority faces the following four scenarios. The first, baseline, scenario corresponds to a situation where the financial authority does not account for climate-related considerations. By definition, this is a scenario of "no coordination" between micro- and macroprudential measures. In this scenario, microprudential tools are set such that $\psi^B = \psi^G$. In this scenario macroprudential policy also ignores climate factors by setting $\theta_e = 0$. In the second scenario, the financial authority aligns (coordinates) only microprudential policies to account for climate factors, thus $\psi^B < \psi^G$ to reduce leverage of the carbon intensive firms. However, macroprudential policy does not coordinate with microprudential policy and ignores climate factors, thus $\theta_e = 0$. We call this scenario of partial coordination as "microprudential coordination", as only microprudential policy is accounting for climate factors. The third scenario is a partial coordination scenario where only macroprudential policy accounts for climate factors by setting $\theta_e > 0$, while microprudential does coordinate with macroprudential policy, thus $\psi^B = \psi^G$. We label this scenario as "macroprudential coordination". In the fourth scenario, which we label "full coordination", both micro and macroprudential policies account for climate-change factors. Therefore, $\psi^B < \psi^G$ and θ $>$ 0. In the next section, we discuss the results of these scenario exercises and assess the normative implications of micro-macroprudential policy coordination in terms of welfare and output volatility.

5 Micro-macroprudential policy coordination and welfare

To assess the welfare implications of micro-macroprudential policies and their coordination over the risky climate transition, we follow the standard approach in the literature [\(Schmitt-](#page-33-12)[Grohe and Uribe,](#page-33-12) [2007;](#page-33-12) [Wolff and Sims,](#page-34-1) [2017\)](#page-34-1), and express welfare costs in steadystate consumption equivalents (or compensating variations), which is the proportion of each period's consumption that the representative household would need to give up in a deterministic world so that its welfare is equal to the expected conditional utility in the stochastic world.

Let \mathbb{W}_0^i denote the welfare in a state of the world where micro-and macroprudential policies follow regime *i* :

$$
\mathbb{W}_0^i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\nu}{1-\nu} \ln \hat{C}^i + \omega \frac{H_t^i}{1+\nu}^{1+\nu} - \psi \frac{N_t^i}{1+\eta}^{1+\eta} \right), \tag{5.1}
$$

where C_t^i , N_t^i , H_t^i denote optimal paths of consumption, labor and health status under regime *i*.

Let c_e denote the welfare costs of adopting an alternative policy regime, $i = h$ relative to the baseline scenario regime. Formally, *c^e* is implicitly defined by

$$
\mathbb{W}_{0}^{h} = \mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \left(\ln \left(1 - c_{e} \right) \left(\hat{C}_{t}^{l} - h \hat{C}_{t-1}^{l} \right) + \omega \frac{H_{t}^{l}}{1+v}^{1+v} - \psi \frac{N_{t}^{l}}{1+\eta}^{1+\eta} \right). \tag{5.2}
$$

We can write:

$$
\mathbb{W}_0^h = \frac{\ln\left(1 - c_e\right)}{1 - \beta} \mathbb{W}_0^l. \tag{5.3}
$$

This approach allows us to obtain welfare costs (if $c_e < 0$) or welfare gains (if $c_e > 0$) of policy regime *i* in steady state percentage consumption equivalents, as follows:

$$
c_e = 1 - \exp\left[\frac{\mathbb{W}_0^h}{\mathbb{W}_0^l} (1 - \beta)\right] \times 100
$$
 (5.4)

Table [4](#page-27-0) reports the results of this exercise. In particular, the table reports the welfare costs adopting each micro-macroprudential policy regime together with values of output volatility in each regime to show the presence of a welfare-volatility trade-off.

	Scenario			
	no coordination	micro	macro	full
Welfare (conditional) (c_e)	-4.057		-3.815 -2.851 -3.923	
Welfare (unconditional) (c_e)	-5.501	-2.401	-2.012	-2.787
Output volatility	0.0696	0.0812	0.0265	0.0217

Table 4: Welfare, volatility and micro-macroprudential coordination

The results show that conditional on the realization of a positive shock to the corporate carbon tax, welfare costs are higher if micro and/or macroprudential policy do not consider climate factors. These regimes, however, are also the ones with lower output volatility. From a climate policy-making perspective, our results suggest a trade-off between short-run output stabilization, which appears more effective under the regime of "full coordination" and welfare improvements, which are greater under "partial coordination". Therefore, the policymaker faces a trade-off between two desirable objectives. It is useful to discuss the key mechanisms underlying these results. First, the quantitative effects of the policy regimes ultimately depend on the non linearity involved in the welfare objective. These effects relate to the (net) welfare impact that the adoption of a policy regime has on the welfare objective, and particularly on aggregate consumption and aggregate employment in a decentralized equilibrium that is inefficient due to the presence of real, nominal and financial distortions. Therefore, the effect on welfare depend on the consumption/employment and the consumption/health trade-off faced by the representative household. Second, welfare losses also relate to impacts stemming from the volatility of consumption, employment, price inflation and wage inflation. Finally, our results crucially depend on the assumptions made in the main text, which involve the specific modeling of carbon taxes, the way carbon tax revenues are distributed, the elasticity parameters regarding the impact of R&D expenditure in the green sector, the parameters related to emission abatement and the emission target. In addition, the model's sensitivity is also affected by the parameters governing the impact of emissions on household utility, as well as the parameter governing the importance of health to the household.

6 Conclusions

Despite broad recognition by academics and policymakers of the importance of accounting for climate-related factors in public policies, the debate on the appropriate micro- and macroprudential policy tools for addressing systemic risk arising from climate transition risk shocks is still an open issue. The divide concerns particularly the the complexity of the design and the calibration of climate-related tools for prudential regulation and their degree of interaction.

In this paper, we developed a climate-DSGE model to shed light on the plausible policy options available to the regulatory authority that intends to include climate factors into its prudential framework while pursuing the institutional mandate. We have shown that, a carbon tax dampens economic activity and emissions as a consequence of its pro-cyclical nature. This result lends support to the dominant consensus that a Pigouvian approach is an effective tool in lowering emissions. In addition, we have also shown that, conditional on the realization of a carbon tax shock, the financial regulator that intends to manage the resulting systemic risk can adapt both micro and macroprudential policy tools. In this respect, micro and macroprudential policies adapted in isolation can dampen amplification stemming from transition risks, reducing business cycle volatility and emissions, while facing moderate welfare costs. However, if micro and macroprudential policies are both adapted to reduce emissions in "full coordination", business cycle stabilization following the transition risk shock can be achieved with lower welfare costs. Therefore, this paper suggests higher micro-macroprudential policy coordination can minimize the trade-off between efficiency and financial stability. Our framework does not explicitly model technological change nor

capacity transformation across sectors. In addition, it could also be expanded to account for higher sectoral and geographical granularity, or adapted to study specific policy cases. We leave this potential exploration as future research avenues.

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A Derivations

A.1 Derivation of the bank enforcement constraint

There is a continuum of banks indexed by *i*. Their liabilities consist of household deposits (D_t^B) and bank capital (S_t^B) . Their assets consist of holdings of brown and green corporate bonds, L_t^B and L_t^G , respectively, with market price Q_t^B and Q_t^G . The balance sheet of bank *i* at time *t* reads as

$$
Q_t^B L_{i,t}^B + Q_t^G L_{i,t}^G = D_{i,t}^B + S_{i,t}^B.
$$
\n(A.1)

The bank is subject to a capital constraint that endogenously limits leverage derived as a weak contract enforcement problem [\(Gertler and Karadi,](#page-32-4) [2011\)](#page-32-4). Therefore, we assume that the bank can abscond with some of its risk-weighted assets and default. In that case, the bank can take a fraction $\theta_t \rho^B Q_t^B L_{i,t}^B$ of brown assets and fraction $\theta_t \rho^G Q_t^G L_{i,t}^G$ of green assets. Depositors can recover the remaining shares $1 - \theta_t \rho^B Q_t^B L_{i,t}^B$ and $\theta_t \rho^G Q_t^G L_{i,t}^G$ for each asset. Contract enforcement requires that for the household to have an incentive to deposit with the bank, the bank's continuation value $(v_{i,t})$ must be at least equal to the fraction of pledgeable assets. Then, the following constraint holds:

$$
v_{i,t} \geq \theta_t \left(\rho^B Q_t^B l_{i,t}^B + \rho^G Q_t^G l_{i,t}^G \right), \tag{A.2}
$$

where ρ^B , $\rho^G > 0$ are the degree of asset pledgeability, which can also be interpreted as riskweights. The term θ_t is an aggregate credit shock.

A.2 Aggregation and market clearing

Aggregate price and wage inflation are given by

$$
1 = (1 - \phi_p) (\Pi_t^*)^{1 - \varepsilon_p} + \phi_p \Pi_t^{\varepsilon_p - 1},
$$
 (A.3)

$$
w_t^{1-\varepsilon_w} = (1-\phi_w)(w_t^*)^{1-\varepsilon_w} + \phi_w \Pi_t^{\varepsilon_w-1} w_{t-1}^{1-\varepsilon_w}.
$$
 (A.4)

Aggregating across retailers, sectors and labor unions delivers:

$$
Y_t^G = v_t^P Y_{r,t}^G \tag{A.5}
$$

$$
Y_t^B = v_t^p Y_{r,t}^B \tag{A.6}
$$

$$
Y_t = Y_{r,t}^B + Y_{rt}^G \tag{A.7}
$$

$$
Y_t^B + Y_t^G = Y_t \tag{A.8}
$$

$$
N_{d,t} = N_{d,t}^B + N_{d,t}^G,
$$
\n(A.9)

$$
N_t = N_{d,t} v_t^w,\tag{A.10}
$$

where v_t^p t_t^p and v_t^w are price and wage dispersion, respectively given by

$$
v_t^p = \phi_p \Pi_t^{\varepsilon_w} v_{t-1}^p + (1 - \phi_p) (\Pi_t^*)^{-\varepsilon_p}, \tag{A.11}
$$

$$
v_t^w = (1 - \phi_w) \left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} + \phi_w \left(\frac{w_t}{w_{t-1}}\right)^{\epsilon_w} \Pi_t^{\epsilon_w} v_{t-1}^w.
$$
 (A.12)

Market clearing in the bond market requires that bonds issues by the the two types of wholesale firms and government, respectively, are held by banks or the central bank:

$$
b_t^G = l_t^G + l_{cb,t}^G.
$$

$$
b_t^B = l_t^B + l_{cb,t}^B.
$$

Finally, budget consolidation leads to the aggregate resource constraint:

$$
Y_t = C_t + I_t + G_t. \tag{A.13}
$$

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