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(UN-)SUSTAINABLE INVESTMENT

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Abstract. To finance the fight against climate change, sustainable investment is projected to surpass $40 trillion by 2030. In principle, sustainable investment diverts funds away from brown firms, increasing their borrowing costs to encourage them to become greener. However, recent empirical evidence does not support this channel, as the most polluting firms tend to become *more* brown in response to higher costs of capital. I formalise this empirical finding by developing a stylised model where brown firms must choose the optimal time to switch from old, polluting technologies to new, clean alternatives. Results indicate that raising the cost of capital for brown firms can have non-monotonic effects on the optimal switching times. For example, firms operating in capital-intensive sectors, often among the largest polluters, are more likely to respond to higher borrowing costs by delaying their switching time. In contrast, brown firms that are nearly ready to switch to cleaner methods may speed up their transition when faced with higher borrowing costs.

JEL Codes: Q50, Q56.

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Résumé Non Technique

L'investissement durable, qui oriente les capitaux vers des entreprises vertes tout en évitant les entreprises qui polluent, a rapidement augmenté sa part de marché. Au niveau global, Bloomberg Intelligence rapporte que les actifs qui respectent les critères environnementaux, sociaux et de gouvernance ont atteint 30 000 milliards de dollars en 2022 et devraient dépasser 40 000 milliards de dollars d'ici 2030. Au niveau de l'Union européenne, la Commission estime que 620 milliards d'euros par an en investissements supplémentaires seront nécessaires pour attendre les objectifs environnementaux en 2030. Selon les estimations de la Banque centrale européenne, fin 2023, la dette durable représentait déjà 7 % des titres de créance émis dans la zone euro.

En principe, la logique de l'investissement durable est simple : en réduisant les investissements dans les entreprises polluantes, elle augmente leur coût de capital, ce qui les incite à devenir plus écologiques. Par conséquent, le succès de l'investissement durable se mesure non seulement par l'augmentation du coût du capital pour les entreprises polluantes, mais aussi par la part de ces entreprises qui deviennent plus vertes en conséquence.

Un nombre croissant d'analyses empiriques trouvent que la première condition de succès est remplie : les investisseurs durables augmentent effectivement le coût du capital pour les entreprises polluantes. Cependant, des études empiriques récentes trouvent que la deuxième condition n'est pas atteinte. Plus précisément, il semble que quand leur coût de capital augmente, les entreprises les plus polluantes deviennent encore plus polluantes.

Dans ce contexte, un modèle simple est présenté pour étudier comment les entreprises polluantes réagissent à l'augmentation de leur coût du capital. Plus précisément, il est étudié sous quelles conditions une entreprise polluante décide de devenir durable. Dans le modèle, l'entreprise choisit à chaque période son niveau d'investissement dans une nouvelle forme de capital écologique, ainsi que le moment auquel elle abandonnera complètement ses méthodes polluantes pour des alternatives durables. Ces décisions visent à minimiser un indice de performance qui capture à la fois le coût de l'accumulation de capital écologique et les coûts

associés à l'abandon définitif des méthodes polluantes. Deux forces concurrentes entrent en jeu.

D'une part, l'augmentation du coût du capital réduit le niveau du stock de capital écologique auquel il devient optimal d'abandonner les pratiques polluantes. Par conséquent, la hausse du coût du capital incite les entreprises polluantes à passer plus tôt à une production écologiquement durable. C'est l'idée qui motive de nombreux investisseurs durables.

D'autre part, la hausse du coût du capital augmente également le coût de l'accumulation de capital écologique. Cela incite l'entreprise à passer plus tard à la production selon des pratiques écologiques. Plus précisément, la hausse du coût du capital réduit le rythme d'accumulation de capital vert. Cette observation est conforme aux principes de base de la finance d'entreprise. Accumuler du capital vert signifie un investissement initial substantiel dont les retours financiers sont reportés dans le futur. Lorsque le coût du capital augmente, les bénéfices à court terme résultant des méthodes polluantes deviennent plus attrayants, décourageant ainsi l'investissement dans des alternatives écologiques. Ce deuxième effet est souvent négligé par les investisseurs durables.

Par conséquent, le modèle étudié prédit que l'augmentation du coût du capital peut avoir des conséquences différentes selon la situation de l'entreprise. Plus précisément, selon les caractéristiques des entreprises polluantes le moment optimal pour basculer vers la nouvelle technologie verte peut se rapprocher ou s'éloigner dans le futur. Par exemple, le modèle suggère que les entreprises opérant dans des secteurs intensifs en capital, où des investissements verts substantiels sont nécessaires pour remplacer les machines, équipements et infrastructures existantes, sont plus susceptibles de réagir à une augmentation de leur coût du capital en retardant le moment de leur transition vers la technologie verte. Les entreprises qui donnent moins de poids aux bénéfices futurs présentent une réponse similaire. En revanche, les entreprises qui ont déjà accumulé un stock de capital écologique sont susceptibles d'accélérer leur transition lorsqu'elles sont confrontées à une hausse de leur coût de capital.

En somme, en accord avec les études empiriques récentes, mon modèle stylisé suggère que l'augmentation des coûts de financement pour les entreprises polluantes pourrait être contre-productive, les amenant à prolonger leur dépendance à des méthodes bien établies mais polluantes. Cela soulève la question cruciale : quel serait le coût optimal du capital pour encourager les entreprises polluantes à passer à des méthodes de production durables ? Aborder cette question constitue une piste intéressante pour de futures recherches, nécessitant probablement une approche en équilibre général.

There is only one true law of history, and that is the law of unintended consequences.

Niall Ferguson

1. Introduction

Sustainable investment, which directs capital towards green firms while avoiding brown ones, has become a growth industry. For example, Bloomberg Intelligence reports that environmental, social, and governance (ESG) assets reached $30 trillion in 2022 and are expected to surpass $40 trillion by 2030.[1] Meanwhile, in the euro area, holdings of sustainable debt securities have grown continuously over the last years, reaching a 7 percent share of total holdings in 2023.[2] Beyond ethical considerations, the rationale behind sustainable investment is straightforward: By withdrawing capital from brown firms, their borrowing costs increase, thus encouraging them to become greener. Therefore, the success of sustainable investment not only depends on its ability to raise the cost of capital for brown firms, but also on brown firms becoming greener as a result.

A growing body of empirical evidence suggests that the first condition for success holds: sustainable investors are indeed raising the cost of capital for brown firms. For instance, Gormsen et al. (2023) find that the average difference in the perceived cost of capital between the greenest and the brownest firms was close to zero before 2016, but has fallen to -2.6 percentage points since then. Also, Bolton and Kacperczyk (2021) and Ľuboš Pástor et al. (2022) document lower expected returns of green firms in capital markets, due to strong increases in investors' environmental concerns. These dynamics result in lower costs of capital for green firms. Furthermore, Kacperczyk and Peydro (2022) reveal that brown firms are compelled to deleverage when their lenders commit to decarbonising their portfolios, suggesting limited substitution to other sources of capital. Along the same lines, Green and Vallée (2024) show that bank divesting policies negatively affect both the financing and operation of coal firms.

However, a recent paper by Hartzmark and Shue (2023) suggests that the second condition for the success of sustainable investment may not be achieved. Specifically, their empirical

[1] See https://www.bloomberg.com/professional/blog/esg-aum-set-to-top-40-trillion-by-2030-anchor-capital-markets/?tactic-page=600488

[2] See https://www.ecb.europa.eu/stats/all-key-statistics/horizontal-indicators/sustainability-indicators/html/index.en.html

evidence shows that the most polluting firms tend to become *more* brown in response to higher costs of capital. In other words, sustainable investing can backfire if its goal is to support the green transition. This insight aligns with Xu and Kim (2022), who show that financial constraints increase firms' toxic emissions. It also echoes the theoretical model in Lanteri and Rampini (2023), where it is optimal for financially constrained firms to invest in old, dirty capital rather than in new, clean capital.

Against this background, I present a stylised model studying how brown firms react to increased capital costs. Specifically, I consider the optimal programming problem of a polluting firm aiming to become a sustainable business. The firm chooses the rate of investment in new green capital per unit of time,[3] and the timing of switching from current polluting methods to sustainable alternatives. These decisions aim to minimise a performance index comprising two terms: a running term and a terminal term.

The running term captures the cost of accumulating green capital per unit of time. Acquiring new green capital often requires significant upfront investment and a departure from existing production techniques. As a result, increasing borrowing costs for brown firms drives up their cost of acquiring green capital, thus penalising this component of the performance index. Instead, the terminal term represents the cost associated with completely abandoning current polluting methods and switching to cleaner alternatives. This terminal term decreases as the firm's green capital stock increases, because brown firms with higher past investments and employee expertise in green production techniques should incur lower transition costs.

To illustrate, in concrete manufacturing, the terminal term could reflect the cost of switching from producing traditional Portland cement to eco-friendly green cement. These costs may include labor productivity losses and stranded assets. However, prior investments in developing low- and zero-carbon concrete technology could facilitate the transition, thereby

[3]Green capital includes the expertise of firm employees in eco-friendly practices as well as the development and accumulation of green technologies and equipment.

reducing these costs. This simple example highlights that the running and the terminal terms are closely related, not independent.

My goal is to characterise analytically how higher capital costs affect the optimal *switching* time - the optimal time to fully transition from current polluting methods to cleaner alternatives. Two competing channels come into play. On the one hand, a higher cost of capital raises brown firms' running costs, thus lowering the optimal level of green capital at the time of the switch. This calls for an earlier adoption of environmentally sustainable measures. This idea motivates many sustainable investors, who tilt their portfolios in favour of green firms, thereby reducing the latter's cost of capital, while simultaneously increasing the cost of capital for excluded brown firms. In principle, these changes in borrowing costs should motivate excluded firms to improve their environmental impacts.

On the other hand, higher borrowing expenses lower the optimal pace of green capital accumulation, leading to a later adoption of environmentally sustainable measures. This is consistent with basic corporate finance. Building up green capital stocks means shifting away from high-pollution production, requiring substantial upfront investments whose associated cash flows are deferred until later. Higher cost of borrowing make short-term profits from polluting methods more appealing, thus discouraging investment in green alternatives. Hartzmark and Shue (2023) suggest that this second channel, often overlooked by sustainable investors, might contribute to their empirical findings.

Hence, the stylised model predicts non-monotonic effects of raising the cost of capital on the optimal switching time. Specifically, brown firms' characteristics, as captured by the structural parameters of the model, determine which channel dominates, and thus whether higher borrowing costs lead to the optimal switching time moving closer or further into the future. For example, the model suggests that firms operating in capital-intensive sectors, where substantial green investments are needed to replace existing brown machinery, equipment, and infrastructure, are more likely to delay their switching times in response to higher borrowing costs. Firms with high discount rates exhibit a similar response. In contrast, firms that are nearly ready to switch to cleaner methods may speed up their transition when faced with higher borrowing costs.

Importantly, I focus on the effects of higher capital costs on the optimal switching time, rather than on the flow of waste material resulting from production processes, including carbon emissions or water pollution. The reason is mathematical tractability. Indeed, the baseline setup belongs to a class of problems known as minimum time problems, for which a closed-form solution is not always easy to find (Bryson, 1975). Restricting attention to the optimal switching time helps in this regard.

Nonetheless, in one of three robustness checks, I extend the model to allow firms to accumulate abatement capital that offsets the flow of environmental degradation per unit of time. This extended model is inspired by the literature on economic growth and the environment reviewed by Xepapadeas (2005). The extended model confirms that the paper's key insight does not result from some idiosyncratic aspect of the baseline model. Indeed, the extended model conveys the same main message: increasing financing costs for brown firms might backfire, causing them to prolong their reliance on well-established, polluting methods. Furthermore, the extended model reveals that firms operating in hard-to-abate sectors are more inclined to delay their switching times in response to higher borrowing costs. This observation aligns with the insight from the baseline model, that firms operating in capital-intensive sectors, often among the largest polluters, are also more likely to react to higher borrowing costs by postponing their adoption of eco-friendly practices.

My model is linked to previous theoretical studies of sustainable investing. For example, Heinkel et al. (2001) present a static model where exclusionary ethical investing results in lower stock prices for polluting firms, thereby increasing their cost of capital and reducing their investment. Albuquerque et al. (2019) develop a model in which a firm's socially responsible investments enhance customer loyalty, thereby reducing its risk and increasing its value. Also, Ľuboš Pástor et al. (2021) construct a one-period model predicting that sustainable investment leads to positive social impact by making brown firms greener and by shifting investment towards green firms. Lastly, Broccardo et al. (2022) examine the effectiveness of exit (divestment and boycott) and voice (engagement) strategies in fostering socially desirable outcomes. They find that engagement tends to be more effective.

In addition, this paper connects to the growing literature using dynamic general equilibrium models to study environmental concerns. In their seminal paper, Bovenberg and Smulders (1995) explore the link between environmental quality and economic growth in an endogenous growth model, emphasising the conditions under which sustainable growth is both feasible and optimal. More recently, Hassler et al. (2016) stress the choice between technologies with different impact on the quality of the environment. Acemoglu et al. (2012) and Acemoglu et al. (2016) develop endogenous growth models with clean and dirty technologies, focusing on the optimal use of carbon taxes and green subsidies. Likewise, Golosov et al. (2014) examine optimal carbon taxes, exploring their sensitivity to key factors, including the discount rate and the economic losses resulting from carbon emissions.

Unlike these papers, I present a tractable, continuous time model assessing the conditions under which raising the cost of capital for brown firms might be counterproductive for the green transition. To the best of my knowledge, this is the first paper drawing attention to how higher borrowing costs could affect the target date at which firms aim to fulfil their sustainability pledges.

The remainder of the paper is organised as follows. Section 2 develops the baseline model and conveys the key messages. Section 3 presents two robustness checks. Section 4 concludes.

2. The baseline model

2.1. **Setup.** Consider the optimal programming problem of a brown firm aiming to become a sustainable business. Let $x(t)$ be a state variable summarising the firm's stock of green capital. This stock might include firm employee expertise in eco-friendly practices or past investments in green technologies and equipment intended to replace current polluting ones at a future date. Variable $x(t)$ evolves according to

$$\dot{x}(t) = u(t), \quad x(0) \geq 0, \tag{1}$$

where $u(t) \geq 0$ is the amount of new green capital produced per unit of time. This production requires borrowed capital (external financing), k. Specifically, suppose a Cobb-Douglas production function $u = \sqrt{2k}$, where I set the exponent to 0.5 to be able to solve the model analytically. Let $r > 0$ be the cost of borrowing capital. The cost function for producing u units of new green capital is then $c(u) = \frac{r}{2}u^2$.

The firm minimises the performance index

$$J = \int_0^\tau [c(u) + \psi]\, dt + g(x(\tau)). \tag{2}$$

The first term in the right hand side represents the running cost over $(0, \tau)$ for the firm until it transitions to sustainable practices. It consists of the cost of accumulating the stock of green capital, $c(u)$, and the costs associated with being perceived as a brown firm by investors and consumers, denoted $\psi > 0$. These costs may include lower market values (Ľuboš Pástor et al., 2021) and reduced profit margins (Albuquerque et al., 2019).[4] As for the terminal cost function $g(x(\tau))$, I assume the following.

Assumption 1. *The terminal cost function $g(x)$: $\mathbb{R}^+ \to \mathbb{R}^+$ features $g_x(x) < 0$, $g_{xx}(x) > 0$ and $g(\infty) = 0$. These properties directly imply $g_x(\infty) = g_{xx}(\infty) = 0$.*

Function $g(x(\tau))$ represents the cost associated with transitioning from existing polluting methods to sustainable ones. Naturally, this cost decreases as the firm's green capital stock increases. For instance, in concrete manufacturing, which is among the highest emitters of CO_2, $g(x(\tau))$ could represent the cost of transitioning from producing traditional concrete, known as Portland cement, to eco-friendly alternatives like green cement. The better equipped the firm is by past investment and expertise of its employees with eco-friendly alternatives, the lower the transition cost.

The firm's programming problem is to find the function $u(t)$ and the switch time τ to minimise the performance index J.

Remark. In this baseline setup, the firm does not discount future costs. Two reasons motivate this assumption. First, mathematical tractability, as setting the discount rate to zero greatly

[4]In reality, $c(u)$ and ψ are probably linked. For simplicity, however, I assume they are independent. This has no bearing on the model's key insights. In addition, I stress that setting $\psi > 0$ is required for the programming problem to have a well-defined interior solution. If $\psi = 0$, then setting $u(t) = 0\ \forall t$ and letting $\tau \to \infty$ would bring the performance index to its minimum achievable value - zero. However, this reasoning does not apply when τ has an exogenous upper bound T, as discussed in subsection 3.1. Nonetheless, even in this case, $\psi > 0$ remains a necessary condition. Therefore, $\psi > 0$ might also be viewed as a technical constant.

simplifies the analysis, providing analytical insights rather than numerical estimates. Second, the pressure to address environmental issues can only increase over time, rendering a scenario where the cost of polluting falls over time less relevant. Nevertheless, subsection 3.2 relaxes the no discounting assumption to show that the model's key insights remain unchanged.

2.2. **Solution.** The above setup is formally known as a minimum time problem. I solve it using the calculus of variations. Define the function H as follows

$$H(u(t), \lambda(t)) = \psi + c(u) + \lambda(t)u(t),$$

where $\lambda(t)$ is the Lagrange multiplier or costate. Then the optimal control function $u^*(t)$ and the optimal switch time τ^* solve the following equations (see e.g. Bryson, 1975)

$$\begin{aligned} \dot{x}^*(t) &= u^*(t), \\ \dot{\lambda}^*(t) &= 0, \\ H_u(u^*(t), \lambda^*(t)) &= 0, \end{aligned}$$

together with the boundary conditions

$$\begin{aligned} x(0) &\geq 0 \text{ given}, \\ \lambda^*(\tau^*) &= g'(x(\tau^*)), \\ H(u^*(\tau^*), \lambda^*(\tau^*)) &= 0. \end{aligned}$$

Importantly, these necessary conditions for a regular control minimum are also sufficient, because $H_{uu}(u^*(t), \lambda^*(t)) = r > 0$. For reasons that will become clear below, I assume the following.

Assumption 2. $g_x^{-1}\left(-\sqrt{2r\psi}\right) \geq x(0)$.

The inverse function g_x^{-1} exists, since Assumption 1 ensures that $g_x(x)$ is one-to-one. The next proposition presents the optimal path of green capital along with the optimal switch time τ^* resulting from solving the two-point boundary value problem stated above.

Proposition 1. *Under Assumptions 1 and 2, the optimal stock of green capital evolves according to*

$$x^*(t) = x(0) + \sqrt{\frac{2\psi}{r}}t,$$

and its terminal value at τ^* is

$$x(\tau^*) = g_x^{-1}\left(-\sqrt{2r\psi}\right) \geq x(0).$$

The optimal switch time is thus

$$\tau^* = \sqrt{\frac{r}{2\psi}}(x(\tau^*) - x(0)) \geq 0.$$

Assumption 2 then ensures $x(\tau^*) \geq x(0)$, so that $\tau^* \geq 0$. Proposition 1 allows me to address this paper's subject: does increasing the cost of capital for brown firms prompt them to switch earlier to environmentally sustainable production? Or does it backfire, pushing them to stick with brown practices, as suggested by recent empirical evidence in Hartzmark and Shue (2023)? I turn to this next.

2.3. **Insights.** When the cost of borrowing capital, r, goes up, two competing channels come into play. On the one hand, a higher r raises the running cost of the firm that has not yet switched to sustainable production. This motivates the firm to adopt green practices sooner; that is, it brings τ^* closer in time. To see this point more clearly, Proposition 1 implies $\frac{\partial x(\tau^*)}{\partial r} < 0$. Hence, higher financing costs lower the optimal level of green capital at the time of the switch, thus calling for an earlier adoption of environmentally sustainable measures. This idea is behind the strategies of most sustainable investors, who aim to encourage companies to become greener by raising the cost of capital for those that are still polluting.

On the other hand, a higher r also raises the cost of accumulating green capital. This motivates the firm to adopt green practices later; that is, it pushes τ^* farther into the future. To see this point more clearly, Proposition 1 implies $\frac{\partial^2 x(t)}{\partial r \partial t} < 0$. Hence, higher costs of funds lower the optimal pace of green capital accumulation, leading to a later adoption of environmentally sustainable measures. This is consistent with basic corporate finance. Building up green capital means shifting away from high-pollution production, requiring substantial upfront investment with associated cash flows deferred until later. When the cost of borrowing rises, it makes short-term profits from polluting methods more appealing, thus discouraging investment in green alternatives. This second channel, often overlooked by sustainable investors, has been stressed in recent empirical research (Hartzmark and Shue, 2023).

My simple model can serve to assess the relative strength of the two channels.

Lemma 1. *Under Assumptions 1 and 2, the optimal switch time τ^* features*

$$\frac{\partial \tau^*}{\partial r} = \frac{1}{2\sqrt{2r\psi}}\left[x(\tau^*) - x(0) + \frac{g_x(x(\tau^*))}{g_{xx}(x(\tau^*))}\right].$$

Therefore,

$$\frac{\partial \tau^*}{\partial r} < 0 \iff x(\tau^*) - x(0) < -\frac{g_x(x(\tau^*))}{g_{xx}(x(\tau^*))}.$$

Proof. Immediate computations from Proposition 1. □

Lemma 1 reveals that the relative strength of the two channels depends on the curvature of the cost function, $g(x)$, near the optimal terminal value $x(\tau^*)$. Suppose, for example, $g(x)$ is almost linear near $x(\tau^*)$. Then, $\frac{g_x(x(\tau^*))}{g_{xx}(x(\tau^*))} \to \infty$, $\frac{\partial \tau^*}{\partial r} < 0$, and the first channel dominates. If instead the terminal cost function is highly convex near $x(\tau^*)$, then $\frac{g_x(x(\tau^*))}{g_{xx}(x(\tau^*))} \to 0$, $\frac{\partial \tau^*}{\partial r} > 0$, and the second channel dominates. To delve deeper into the analysis, assume the following.

Assumption 3. *The terminal cost function $g(x)$ features*

$$\frac{g_x(x)}{g_{xx}(x)} = -\gamma_1 - \gamma_0 x,$$

where $\gamma_0 \in [0, 1)$ and $\gamma_1 > 0$.

While Assumption 3 may appear arbitrary at first glance, it is consistent with most standard functions meeting the conditions imposed on $g(x)$. For example, $g(x) = ae^{-x/b}$ with $a, b > 0$ implies $\gamma_1 = b$ and $\gamma_0 = 0$; $g(x) = \frac{c}{a+bx}$ with $a \geq 0$ and $b, c > 0$ implies $\gamma_1 = \frac{a}{2b}$ and $\gamma_0 = \frac{1}{2}$; and $g(x) = \frac{a}{x^b}$ with $a, b > 0$ implies $\gamma_1 = 0$ and $\gamma_0 = \frac{1}{1+b}$.

Lemma 2. *Under Assumption 3, the function*

$$f(x) = x - x(0) + \frac{g_x(x)}{g_{xx}(x)}$$

has a unique root given by

$$\bar{x} = \frac{x(0) + \gamma_1}{1 - \gamma_0}.$$

Moreover, $f(x) < 0 \; \forall x \in (0, \bar{x})$ *and* $f(x) > 0 \; \forall x \in (\bar{x}, \infty)$*.*

Proof. Under Assumption 3, $f(0) < 0$, $\lim_{x \to \infty} f(x) = \infty$ and $f_x(x) > 0$ everywhere. $\square$

Combining Lemmas 1 and 2 provides the paper's main insight.

Proposition 2. *Under Assumptions 1, 2 and 3, the optimal switch time* τ^* *features*

$$\frac{\partial \tau^*}{\partial r} \begin{cases} < 0 & \text{if } x(\tau^*) < \bar{x}, \\ = 0 & \text{if } x(\tau^*) = \bar{x}, \\ > 0 & \text{if } x(\tau^*) > \bar{x}. \end{cases}$$

Therefore, raising the cost of funds for brown firms has its desired effect if and only if the optimal level of green capital at the time of the switch, $x(\tau^*)$, lies below the threshold $\bar{x}$. If this condition is not met, the policy is counterproductive, as the second channel dominates. Factors favouring $x(\tau^*) < \bar{x}$ include: a high cost of being perceived as a brown firm, because $\frac{\partial x(\tau^*)}{\partial \psi} < 0$; an already high cost of external funds, because $\frac{\partial x(\tau^*)}{\partial r} < 0$; a substantial initial stock of green capital, because $\frac{\partial \bar{x}}{\partial x(0)} > 0$; and a low transition cost from polluting methods to sustainable ones. To better illustrate the latter, consider a simple example: $g(x) = ae^{-x/b}$ with $a, b > 0$, yielding $x(\tau^*) = -b\log\left[\frac{b\sqrt{2r\psi}}{a}\right]$ and $\bar{x} = x(0) + b$. Clearly, higher a's lead to higher transition costs. Since $\frac{\partial x(\tau^*)}{\partial a} > 0$ and $\frac{\partial \bar{x}}{\partial a} = 0$, reducing a makes $x(\tau^*) < \bar{x}$ more likely.[5]

Continuing with the example $g(x) = ae^{-x/b}$, we should expect that firms in high capital-intensive industries, such as construction, transportation, utilities and manufacturing, feature higher a's than firms in low capital-intensive industries, such as education, software and technology, financial services and healthcare. The reason is that the cost of transitioning to sustainable methods with a low stock of green capital must be higher in industries that are heavily reliant on machinery, equipment, and infrastructure. As a result, Proposition 3

[5]The same logic holds for the other examples of the function $g(x)$ provided above.

implies that an increase in financing costs is more likely to backfire in high capital-intensive sectors, which tend to be the biggest polluters.

A complementary way of presenting Proposition 3 is to characterise $\frac{\partial \tau^*}{\partial r}$ not as a function of $x(\tau^*)$, but as a function of r.

Corollary 1. *Define* $\bar{r} = \frac{(g_x(\bar{x}))^2}{2\psi}$, *under which* $x(\tau^*) = \bar{x}$. *Under Assumptions 1, 2 and 3, the optimal switch time* τ^* *features*

$$\frac{\partial \tau^*}{\partial r} \begin{cases} > 0 & \text{if } r < \bar{r}, \\ = 0 & \text{if } r = \bar{r}, \\ < 0 & \text{if } r > \bar{r}. \end{cases}$$

Corollary 1 hammers home the non-monotonic effects of higher financing costs on the optimal switch time, revealing an inverted-U shape of τ^* as a function of r. To the left of $\bar{r}$, small increases in r backfire, as the second channel described at the beginning of this section dominates. In contrast, to the right of $\bar{r}$, increases in r have the intended consequences, as higher running costs encourage the firm to switch sooner to sustainable production.

Returning to the example $g(x) = ae^{-x/b}$, we have $\bar{r} = \frac{a^2}{2b^2\psi} e^{-2\left(\frac{x(0)}{b}+1\right)}$, and hence, $\frac{\partial \bar{r}}{\partial a} > 0$. In words, in industries with high transition costs, a greater increase in borrowing costs will be needed to advance the switching time.

2.4. **Summary.** The paper's main insight is this: *increasing financing costs for brown firms might backfire, causing them to prolong their reliance on well-established, polluting methods*. This scenario is especially likely for firms operating in capital-intensive sectors, where substantial green investments are needed to replace existing brown machinery, equipment, and infrastructure. This theoretical insight nicely aligns with the empirical findings of Hartzmark and Shue (2023). Further research is warranted, because capital-intensive sectors tend to be among the largest polluters, while also providing essential goods and services.

In my stylised model, one policy with unambiguous effects involves increasing the costs associated with being perceived as a brown firm ($\Delta\psi > 0$), while holding the cost of borrowing constant. Indeed, it is straightforward to show that $\frac{\partial \tau^*}{\partial \psi} < 0$ everywhere. This makes sense, since parameter ψ does not affect the cost of borrowing, and as a result, only the first channel comes into play. In reality, however, penalising brown firms without raising their cost of funds seems challenging. As mentioned earlier, investors divesting from polluting companies or consumers boycotting their products are likely to raise their borrowing costs; see e.g. Ľuboš Pástor et al. (2022), Kacperczyk and Peydro (2022) and Gormsen et al. (2023).

3. Robustness checks

In this section, I will introduce three extensions to the baseline model discussed so far. These extensions serve two purposes: first, to demonstrate the robustness of the model's main messages; and second, to uncover additional insights.

3.1. **Maximum switch time.** So far, I have assumed an infinite horizon where $\tau^* \in (0, \infty)$. In reality, however, this might be a strong assumption, as many firms operate within jurisdictions with binding commitments to halt environmental degradation by a certain date. For instance, the European Union aims to achieve climate neutrality by 2050, with this objective enshrined in the European Climate Law.

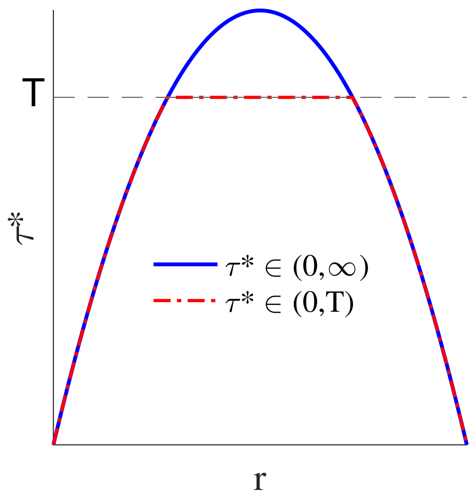
Therefore, let me introduce a maximum switch time $T < \infty$, representing the deadline by which the firm must adopt environmentally sustainable methods if it has not already done so. Formally, the firm chooses the function $u(t)$ and the switch time τ to minimize (2) subject to (1) and the inequality constraint $\tau < T$. The sufficient conditions for a regular control minimum are almost identical to those presented earlier. The only difference is that the transversality condition $H(u^*(\tau^*), \lambda^*(\tau^*)) = 0$ becomes $H(u^*(\tau^*), \lambda^*(\tau^*))(\tau^* - T) = 0$, which is a standard slackness condition.

Hence, if τ^* in Proposition 1 is lower than T, the inequality constraint does not bind, and the solution remains unchanged. However, if τ^* is higher than T, then the constraint binds and the pair $(\tau^*, x(\tau^*))$ reported in Proposition 1 is no longer admissible. Instead, $\tau^* = T$ and $x(\tau^*)$ can be directly obtained from its law of motion: $x(\tau^*) = x(0) + \sqrt{\frac{2\psi}{r}}T$. As a result, firms constrained by the deadline exhibit

$$\frac{\partial \tau^*}{\partial r} = 0 \quad \frac{\partial x(\tau^*)}{\partial r} < 0.$$

Figure 1 provides a graphical illustration, highlighting how a maximum switch time helps mitigate the counterproductive effects of increasing borrowing costs for brown firms. To see

FIGURE 1. Optimal switch time as a function of borrowing costs



Notes. Solid blue line represents the baseline model presented in Section 2. Dashed red line represents the maximum switch time extension. Corollary 1 derived the inverted-U shape of τ^* as a function of r.

this point more clearly, consider a continuum of firms with some degree of heterogeneity (for example, a in $g(x) = ae^{-x/b}$ might be drawn from a continuous distribution function $G(a)$), leading to a distribution of optimal switch times. Following an increase in r, some firms will increase their τ^*, while others will decrease them. However, by imposing an upper limit on these τ's, as depicted by the horizontal segment of the dashed red line in Figure 1, a maximum switch time alleviates the unintended effects of a higher r.

3.2. **Discounting and modified law of motion.** I now relax the no-discounting assumption and also allow the current stock of green capital to help generate new capital. Formally, the firm's programming problem is

$$\begin{array}{ll} \text{Minimise} & \int_0^\tau e^{-\rho t}\left[c(u)+\psi\right]dt + e^{-\rho\tau}g(x(\tau)), \\ \text{subject to} & \dot{x}(t) = u(t) + ax(t), \quad x(0) \geq 0, \end{array}$$

where $\rho \geq 0$ is the discount factor; and $a \geq 0$ determines the growth rate of $x(t)$ in the absence of new production $u(t)$. Setting $\rho = a = 0$ brings us back to the baseline model.

The solution method is exactly as before. It can be shown that the optimal terminal level of green capital, $x(\tau^*)$, is a root of the nonlinear function

$$m(x) = \psi + g_x(x)\left[ax - \frac{g_x(x)}{2r}\right] - \rho g(x).$$

To ensure this root exists and is unique, I assume the following.

Assumption 4. *$\rho \geq a$, and $\psi < \frac{g_x(0)^2}{2r} + \rho g(0) - \lim_{x\to 0} a g_x(x)x$.*

Under Assumption 4, $\lim_{x\to 0} m(x) < 0$, $m_x(x) > 0$ and $\lim_{x\to\infty} m(x) > 0$. Therefore, $m(x)$ has a unique root, denoted by $\bar{x}$, providing the optimal terminal level of green capital. That is, $x(\tau^*) = \bar{x}$. Unfortunately, since $\bar{x}$ is only implicitly defined, I cannot provide parameter restrictions ensuring $x(\tau^*) > x(0)$, as Assumption 2 does in the baseline model. Therefore, what follows implicitly assumes $x(\tau^*) > x(0)$.

As for the optimal switch time, τ^*, it is a root of the nonlinear function

$$n(\tau) = \bar{x} - x(0) - a\bar{x}\tau + \frac{1}{\rho - a}\frac{g_x(\bar{x})}{r} + \frac{1}{a-\rho}\frac{g_x(\bar{x})}{r}e^{(a-\rho)\tau}.$$

Since $\lim_{\tau\to 0} n(\tau) > 0$, $n_\tau(\tau) < 0$ and $\lim_{\tau\to\infty} n(\tau) = -\infty$, this equation has a unique positive root, $\bar{\tau}$. Hence, $\tau^* = \bar{\tau}$.

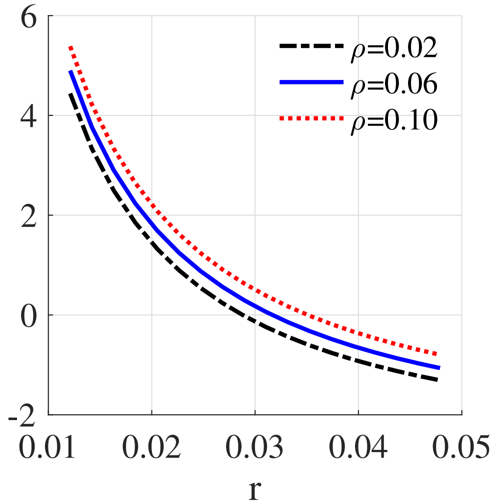
In this extended version of the model, I have $\frac{\partial x(\tau^*)}{\partial r} < 0$, as in the baseline version (see Lemma 1). However, the analytical expression for $\frac{\partial \tau^*}{\partial r}$ is lengthy and not particularly enlightening. Hence, I illustrate its features numerically. Let me assume $g(x) = 0.2e^{-x}$, $a = 0.02$, $\psi = 0.1$ and $x(0) = 0$, satisfying the required assumptions 1 and 4. Needles to say, this parametrisation is arbitrary, and alternative ones can be used without affecting the discussion below.

As Figure 2 shows, increasing borrowing costs can still backfire, delaying the adoption of sustainable practices. Specifically, Figure 2 is the numerical counterpart of Corollary 1, stressing the inverted-U shape of τ^* as a function of r.

Figure 2 also shows that under a higher discount rate, increasing borrowing costs backfires for a broader range of interest rates. This occurs because higher discount rates strengthen the second channel discussed in subsection 2.3. Recall that a higher r makes it more costly to accumulate green capital, which leads to later adoption of green practices. A higher discount rate strengthens this channel by lowering the weight of the terminal cost function in the performance index, J, and increasing the weight of the running cost function.

3.3. **Environmental degradation and abatement capital.** This final subsection establishes that the paper's key insight cannot be attributed to idiosyncratic features of the underlying setup. To this end, I present a slightly different model, which is closer to the literature on

FIGURE 2. Partial derivative $\frac{\partial \tau^*}{\partial r}$ as a function of r



Notes. The vertical axis represents $\frac{\partial \tau^*}{\partial r}$. These partial derivatives are obtained under the following parametrisation $g(x) = 0.2e^{-x}$, $a = 0.02$, $\psi = 0.1$ and $x(0) = 0$.

economic growth and the environment reviewed by Xepapadeas (2005). Crucially, the main message remains unchanged: increasing financing costs for brown firms might backfire, causing them to prolong their reliance on well-established, polluting methods.

Consider a brown firm generating environmental pollution, $p(t)$, as a by-product of its production process. For simplicity, assume the rate of new environmental degradation per unit of time remains constant, independent of the firm's output level. As in Bovenberg and Smulders (1995), however, the firm can accumulate abatement capital to lower its environmental footprint. The stock of pollution generated by the firm then evolves according to

$$\dot{p}(t) = \phi - \psi x(t), \; p(0) \geq 0, \tag{4}$$

where ϕ is the rate of new pollution per unit of time, and ψ governs the effectiveness of existing abatement capital, $x(t)$, in mitigating environmental degradation.

The firm allocates its effort, normalised to one unit each instant, between two activities: producing output, $y(t)$, and creating new abatement capital, $u(t)$. Output is sold in a competitive market at a fixed price normalised to one, while new abatement capital adds to the existing stock according to

$$\dot{x}(t) = u(t), \; x(0) \geq 0 \text{ given.} \tag{5}$$

Both $y(t)$ and $u(t)$ are produced using effort and borrowed capital, through a Leontief technology.[6] The input requirements for a unit of $y(t)$ ($u(t)$) are one unit of effort and γ (α) units of borrowed capital. Formally, the production functions are

$$y(t) = \min\left(m(t), \frac{k_y(t)}{\gamma}\right), \tag{6}$$

$$u(t) = \min\left(1 - m(t), \frac{k_u(t)}{\alpha}\right), \tag{7}$$

where $m(t) \in (0,1)$ is the effort allocated to produce $y(t)$, and $k_y(t)$, $k_u(t)$ are the amounts of borrowed capital used to produced $y(t)$ and $u(t)$, respectively. As before, let $r > 0$ be the cost of borrowing capital. Then, the firm's profits per unit of time are

$$\pi(t) = y(t) - r\left(k_y(t) + k_u(t)\right). \tag{8}$$

The firm minimises the performance index

$$J = \int_0^\tau \frac{(\pi(t) - \bar{\pi})^2}{2} dt, \tag{9}$$

where $\bar{\pi} > 0$. In other words, it seeks to minimise the deviation of its profits from a target level $\bar{\pi}$. This objective function, being quadratic, transforms the firm's optimisation program into a linear quadratic control problem, leading to a closed-form solution.

The terminal time τ is not predetermined but is instead a control variable to be selected along with the control functions $\{m(t), k_y(t), k_u(t)\}$ to minimise J while satisfying the 'net-zero' constraint

$$\dot{p}(\tau) = 0. \tag{10}$$

This constraint states that at the terminal time τ, the firm's environmental impact must be zero. For example, suppose the firm's environmental degradation, ϕ, represents greenhouse gas emissions. Then, $\dot{p}(\tau) = 0$ indicates that at the end of the time horizon, the greenhouse gases the firm releases into the atmosphere are balanced by those it removes through its abatement technology. In reality, most firms are making commitments to not add to the total

[6]Leontief technology is required for a closed-form solution of the model, as in any equilibrium effort and borrowed capital are used in exactly the same proportion; see Luttmer (2019) for a similar assumption.

amount of greenhouse gases in the atmosphere.[7] While some firms are making these commitments on their own initiative, others are submitting to outside pressure from investors, employees and governments. Either way, the intuition behind the 'net-zero' constraint (10) is clear.

In sum, the firm's programming problem is to find functions $\{m(t), k_y(t), k_u(t)\}$ and the switch time τ to minimise the performance index J, given the law of motions for $p(t)$ and $x(t)$, the production functions for $y(t)$ and $u(t)$, and the net-zero constraint.

As before, this new setup is a minimum time problem. Therefore, I solve it using the calculus of variations. Appendix A provides the details. Importantly, the problem is well-defined only if two conditions are met: (i) $\tau^* > 0$; and (ii) $m^*(t) \in (0, 1)$. As will become clear shortly, the following parameter restrictions ensure (i)-(ii) are satisfied.

Assumption 5. $2 + r(\alpha - 2\gamma) > \bar{\pi} > 1 - r\gamma$*, and* $\phi > \psi x(0)$.

In addition, above I mentioned the point by Hartzmark and Shue (2023) that developing green technologies and capital often requires larger investments than running existing brown technology. Therefore, I assume the following.

Assumption 6. $\alpha > \gamma$.

In words, producing one unit of new abatement capital, $u(t)$, requires more borrowed capital than producing one unit of final output, $y(t)$.

The next proposition presents the optimal paths for $\{m^*(t), k_y^*(t), k_u^*(t)\}$ along with the optimal switch time τ^* characterising the solution of the minimum time problem.

Proposition 3. *Under Assumptions 5 and 6, the optimal effort allocated to producing y is*

$$m^*(t) = m^* = \frac{2 - \bar{\pi} + r(\alpha - 2\gamma)}{1 + r(\alpha - \gamma)} \in (0, 1),$$

and the optimal levels of borrowed capital are

$$k_y^*(t) = \gamma m^*,$$

$$k_u^*(t) = \alpha(1 - m^*).$$

[7] According to the Net Zero Tracker, for example, half of the world's largest companies are committed to reaching net zero in the coming decades.

The optimal switching time is thus

$$\tau^* = \frac{\phi/\psi - x(0)}{1 - m^*} > 0.$$

Proof. See Appendix A. □

Proposition 3 allows me to shed further light on this paper's subject: does increasing the cost of funds for brown firms prompt them to switch sooner to environmentally sustainable methods?

Proposition 4. *Under Assumptions 5 and 6, the optimal effort allocated to producing $y(t)$ and the optimal switching time feature*

$$\frac{\partial m^*}{\partial r}, \frac{\partial \tau^*}{\partial r} \begin{cases} < 0 & \text{if } \bar{\pi} < \frac{\alpha}{\alpha-\gamma}, \\ = 0 & \text{if } \bar{\pi} = \frac{\alpha}{\alpha-\gamma}, \\ > 0 & \text{if } \bar{\pi} > \frac{\alpha}{\alpha-\gamma}. \end{cases}$$

Therefore, as in the baseline model, this extended setup also suggests non-monotonic effects of increasing borrowing costs on the optimal switching time. Specifically, whether raising r brings τ^* forward or backward depends on the relative sizes of three parameters: the target level for profits, and the capital inputs required to produce one unit of $u(t)$ and $y(t)$.

To grasp the economic intuition, consider a simple thought experiment. Imagine a brown firm initially allocates its effort to generate new abatement capital and to produce final output so that its profits, $\pi(t)$, match their target level $\bar{\pi}$. Moreover, suppose this firm has a value of α significantly higher than γ, indicating that generating new abatement capital requires borrowing much larger amounts than producing final output. Now, assume borrowing costs increase, leading to lower profits and creating a gap between $\pi(t)$ and $\bar{\pi}$. In response to higher borrowing costs, it becomes optimal to allocate more effort to producing final output $y(t)$. Indeed, this adjustment increases the firm's revenue and reduces its overall borrowing costs (i.e. $r\left(k_y(t) + k_u(t)\right)$), since producing $y(t)$ requires less borrowing than generating $u(t)$. Therefore, allocating more effort to produce $y(t)$ narrows the gap between $\pi(t)$ and $\bar{\pi}$, lowering the performance index J. However, allocating more effort to produce $y(t)$ also implies that the net-zero constraint is reached later, thereby delaying the optimal switch time. Then, in this case, $\frac{\partial \tau^*}{\partial r} > 0$.

Suppose the same scenario as before, but now α approaches γ from above. In this case, allocating more effort to produce $y(t)$ in response to an increase in r does not significantly reduce the firm's overall borrowing costs. This is because the borrowed capital requirements in both sectors are similar. Therefore, raising production of new abatement capital becomes optimal, and the firm reaches the net-zero constraint earlier, thus lowering the performance index. Then, in this case, $\frac{\partial \tau^*}{\partial r} < 0$.

All told, the extended model confirms the robustness of the baseline model presented in Section 2. More precisely, it stresses that brown firms' characteristics, as captured by the structural parameters of the model, determine whether higher borrowing costs lead to earlier or later adoption of sustainable production methods. Furthermore, it reveals that firms operating in hard-to-abate sectors (i.e. sectors featuring high α's) are more inclined to delay their switching times in response to higher borrowing costs. This observation confirms the insight from the baseline model that firms operating in capital-intensive sectors, often among the largest polluters, are also more likely to respond to higher borrowing costs by postponing their adoption of eco-friendly practices.

4. CONCLUDING REMARKS

In line with recent empirical evidence, my stylised model suggests that increasing financing costs for brown firms might backfire, causing them to prolong their reliance on well-established, polluting methods. This raises the crucial question: what would be the optimal cost of capital to encourage brown firms to switch to sustainable production methods? Addressing this question is an interesting avenue for future research, probably requiring a general equilibrium approach.

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Appendix A. Solution of the model with abatement capital

As mentioned in the main text, the extended model with environmental degradation and abatement capital is a minimum time problem. I solve it using the calculus of variations. Because of Leontief technology, the following linear equations hold at any equilibrium

$$
\begin{aligned}
k_u(t) &= \alpha(1 - m(t)),\\
k_y(t) &= \gamma m(t),\\
u(t) &= 1 - m(t),\\
y(t) &= m(t).
\end{aligned}
$$

Hence, the stock of abatement capital evolves by

$$\dot{x}(t) = 1 - m(t).$$

Define the function H as follows

$$H(m(t), \lambda(t), \mu(t)) = 0.5(\chi m(t) - r\alpha - \bar{\pi})^2 + \lambda(t)(\phi - \psi x(t)) + \mu(t)(1 - m(t)),$$

where $\lambda(t)$ and $\mu(t)$ are Lagrange multipliers and $\chi = 1 + r(\alpha - \gamma)$. Then the optimal control function $m^*(t)$ and the optimal switch time τ^* solve the following equations (see e.g. Bryson, 1975)

$$
\begin{aligned}
\dot{x}^*(t) &= 1 - m^*(t), \\
\dot{p}^*(t) &= \phi - \psi x^*(t), \\
\dot{\lambda}^*(t) &= 0, \\
\dot{\mu}^*(t) &= \lambda(t)\psi, \\
H_m(m(t), \lambda(t), \mu(t)) &= 0,
\end{aligned}
$$

together with the boundary conditions

$$
\begin{aligned}
x(0) &\geq 0 \text{ given}, \\
x^*(\tau^*) &= \frac{\phi}{\psi}, \\
p(0) &\geq 0 \text{ given}, \\
\lambda^*(\tau^*) &= 0, \\
H(m^*(t), \lambda^*(t), \mu^*(t)) &= 0.
\end{aligned}
$$

Importantly, these necessary conditions for a regular control minimum are also sufficient, because $H_{mm}(m^*(t), \lambda^*(t), \mu^*(t)) = \chi^2 > 0$. Solving this two-point boundary value problem under Assumptions 5 and 6 leads to Proposition 3.

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